Math 9 Fall 19 Homework 4 (Due on Oct 16 before class)

(1) (3 pts) Consider the curve $\mathbf{r}(t) =< 2t^2, t^3, -4t >$, where t is any real number. Find all point(s) on the curve such that the tangent vector of $\mathbf{r}(t)$ is parallel to the plane $10x + 3y - 4z = 7$.

(2) (3 pts each) Consider the curve $\mathcal{C} : \mathbf{q}(t) =< t^2, 2\ln(t), \sqrt{8}t >$ defined on all $t \geq 1$.
   (a) Find parametric equations of the tangent line to the curve $\mathcal{C}$ at the point $(1, 0, \sqrt{8})$.
   (b) Find the arc length of the curve from $(1, 0, \sqrt{8})$ to $(4, 2\ln(2), 2\sqrt{8})$.

(3) (3 pts each) In this question, use product rules for vector valued functions on Stewart p. 898 to solve the two parts below.
   (a) Let $\mathbf{u}(t)$ be a differentiable curve in a sphere. Show that the derivative function $\mathbf{u}'(t)$ is always orthogonal to $\mathbf{u}(t)$. (Solution to this part is actually in the textbook! Now, try to produce an argument for the next part.)

   (b) Define the acceleration function of $\mathbf{u}(t)$ to be the second derivative of $\mathbf{u}(t)$. Let $\mathbf{u}(t)$ be a vector valued function with constant speed, i.e., $|\mathbf{u}'(t)|$ is constant. Show that $\mathbf{u}'(t)$ is always orthogonal to the acceleration function of $\mathbf{u}(t)$ on the points where the acceleration function is not zero.

(4) (3 pts each) Find and sketch the domain and the range for the following functions of two variables.
   (a) $f(x, y) = \sqrt{x^2 - y + 2}$
   (b) $f(x, y) = \ln(x - 1) + \sqrt{x + y}$

(5) (3 pts each) Sketch at least four level curves for each the following function of two variables. Label the constants you used for each level curve you sketch.
   (a) $f(x, y) = 1 + x + y$.
   (b) $f(x, y) = \dfrac{1}{x^2 + y^2}$.