Math 9 Fall 19 Homework 8 (Due on Nov 6 before class)

(1) (3 pts) Linear Algebra 6.4.3

(2) (3 pts) Suppose that $z = 2x^2y + 4x$, $x = s + t - u$, $y = s + t^2$. Find $\frac{\partial z}{\partial s}$, $\frac{\partial z}{\partial t}$, and $\frac{\partial z}{\partial u}$ when $s = t = u = 1$. Your final answers should be numbers.

(3) (3 pts) Let $f(x, y, z) = z \ln x^2 y$.
  (a) Find the maximum and minimum rate of change of $f(x, y, z)$ at $(1, 1, 3)$.
  (b) In what direction does f have the maximum rate of change?
  (c) In what direction does f have the minimum rate of change?

(4) (3 pts) Find points on the ellipsoid

$$\frac{x^2}{4} + y^2 + z^2 = 3$$

such that the tangent plane to the ellipsoid at the point is parallel to

$$3x + 2y + 2z = 3.$$

(5) (6 pts) Let $\mathcal{S}_1$ be the paraboloid $z = x^2 + y^2$ and $\mathcal{S}_2$ be the ellipsoid $4x^2 + y^2 + z^2 = 9$.
  (a) Verify that $(-1, 1, 2)$ is on $\mathcal{S}_1$ and $\mathcal{S}_2$.
  (b) Find the tangent plane P_1 to $\mathcal{S}_1$ at $(-1, 1, 2)$ and the tangent plane P_2 to $\mathcal{S}_2$ at $(-1, 1, 2)$.
  (c) Let $\mathcal{C}$ the curve of intersection of $\mathcal{S}_1$ and $\mathcal{S}_2$. Sketch the curve $\mathcal{C}$.
  (d) Explain why the tangent line to $\mathcal{C}$ at $(-1, 1, 2)$ is actually the intersection of P_1 and P_2.
  (e) Use previous parts to find parametric equations to $\mathcal{C}$ at $(-1, 1, 2)$.

(6) (3 pts each) Find critical points of the following functions and determine whether each of the critical points is a local max, a local min, a saddle, or undetermined.
  (a) $f(x, y) = 2 - y^4 + 2y^2 - 4x^2$
  (b) $g(x, y) = (x^2 + 2y - y^2)e^{-x}$