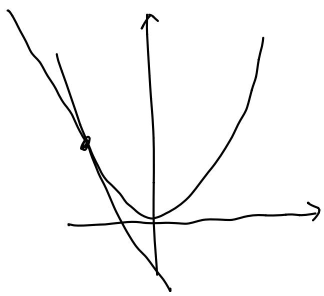
Math 9 F19 Quiz 2 **Name:** Solution

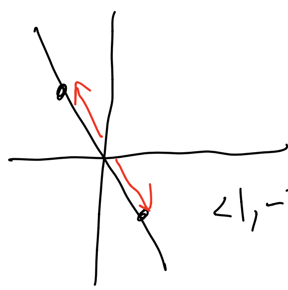
(1) (5 pts) Find *all* unit vectors that are parallel to the tangent line of the parabola $y = x^2$ at the point $(-1, 1)$.



First, find the slope.

Let $y = f(x) = x^2$. $f'(x) = 2x$ and $f'(-1) = -2$.

Next, find a line of slope -2 passing through the origin, which is $y = -2x$.



$\langle 1, -2 \rangle$

$\vec{u} = \langle 1, -2 \rangle$ is a vector parallel to the tangent line.

$$\frac{\vec{u}}{|\vec{u}|} = \frac{\langle 1, -2 \rangle}{\sqrt{1^2 + (-2)^2}} = \frac{\langle 1, -2 \rangle}{\sqrt{5}}$$

is a vector, the other is

$$\frac{\langle -1, 2 \rangle}{\sqrt{5}}.$$

(2) (5 pts) Let $A = \{<3, 1, 1>, <1, 2, 1>, <9, 8, 0>\}$.

(a) Determine the dimension of span(A).

First, there exists a pair of non-parallel vectors $\langle 3, 1, 1 \rangle$ and $\langle 1, 2, 1 \rangle$.

Hence span(A) is not a line.

Now, we determine whethere A is coplanar or not.

Find s, t such that

$$s \langle 3, 1, 1 \rangle + t \langle 1, 2, 1 \rangle = \langle 9, 8, 0 \rangle.$$

$$\left[\begin{array}{cc|c} 3 & 1 & 9 \\ 1 & 2 & 8 \\ 1 & 1 & 0 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 1 & 2 & 8 \\ 3 & 1 & 9 \end{array}\right] \xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2 - R_1} \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 1 & 8 \\ 0 & -2 & 9 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 + 2R_2} \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 1 & 8 \\ 0 & 0 & 25 \end{array}\right]$$

the system is inconsistant.

Therefore, A is not coplanar.

Span $(A) = \mathbb{R}^3$.

(b) Find a basis for span(A).

Because there exist three vectors in A, and span(A) is three-dimensional, A is a basis.