Math 9 F19 Quiz 6 **Name:** Solution

(1) (5 pts) Let the x-axis point east, let the y-axis point north, and let z denotes elevation. Let $f(x, y) = x\sqrt{y}$ and view the graph $z = f(x, y)$ as a mountain. Suppose that you are hiking due south on this mountain through the point $(2, 1, 2)$. Find the tangent line to your path at this point.

South $= \langle 0, -1 \rangle$.

Hence the path is given by

$$\vec{r}(t) = \langle 2 + 0t, 1 - t, f(2 + 0t, 1 - t) \rangle$$

$$= \langle 2, 1 - t, 2\sqrt{1 - t} \rangle$$

$$\vec{r}\,'(0) = \langle 0, -1, \frac{d}{dt}\Big|_{t=0} 2\sqrt{1-t} \rangle$$

$$= \langle 0, -1, \frac{1}{2}\frac{2}{\sqrt{1-t}} \cdot (-1)\Big|_{t=0} \rangle$$

$$= \langle 0, -1, \frac{-1}{\sqrt{1-t}}\Big|_{t=0} \rangle = \langle 0, -1, -1 \rangle.$$

The tangent line is

$$x = 2 + 0t$$
$$y = 1 - t$$
$$z = 2 - t$$

(2) (5 pts) Let S be a surface $\mathbb{R}^3$. You know that the curves

$$\mathbf{r}_1(t) = < 2 + 3t, 1 - t^2, 3 - 4t + t^2 >$$

$$\mathbf{r}_2(u) = < 1 + u^2, 2u^3 - 1, 2u + 1 >$$

both lie on S. Find a scalar equation of the tangent plane to the surface at $P(2, 1, 3)$.

$\vec{r}_1(t) = \langle 2, 1, 3 \rangle$ when $t = 0$. Hence a tangent vector in the plane is

$$\vec{r}_1{}'(0) = \langle 3, -2t, -4 + 2t \rangle \big|_{t=0}$$

$$= \langle 3, 0, -4 \rangle$$

$\vec{r}_2(u) = \langle 2, 1, 3 \rangle$ when $u = 1$

Hene another tangent vector in the plane is

$$\vec{r}_2{}'(1) = \langle 2u, 6u^2, 2 \rangle \big|_{u=1}$$

$$= \langle 2, 6, 2 \rangle$$

A normal vector to the plane is $\vec{r}_1{}'(0) \times \vec{r}_2{}'(1)$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 0 & -4 \\ 2 & 6 & 2 \end{vmatrix} = \langle 24, -14, 18 \rangle$$

scalar eq: $24(x-2) - 14(y-1) + 18(z-3) = 0.$