Math 9 F19 Quiz 7 **Name:** Solution

(1) (10 pts) Let

$$R(x,y) = \begin{bmatrix} 2x+y \\ x-y \\ 2x \end{bmatrix}, S(x,y) = \begin{bmatrix} x+y \\ x-y \end{bmatrix}, T(x,y,z) = \begin{bmatrix} x+y+z \\ y-z \end{bmatrix}$$

be three linear transformations. Determine if the following compositions make sense. *If the composition make sense, find the representing matrix for the composition.*

(a) $R \circ S$

$R : \mathbb{R}^2 \to \mathbb{R}^3$, $S : \mathbb{R}^2 \to \mathbb{R}^2$

$R \circ S$ is well-defined.

$$[R \circ S] = [R][S]$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 0 & 2 \\ 2 & 2 \end{bmatrix}$$ is the rep. matrix.

(b) $S \circ R$

$S \circ R$ is not defined.

$$R(x,y) = \begin{bmatrix} 2x+y \\ x-y \\ 2x \end{bmatrix}, S(x,y) = \begin{bmatrix} x+y \\ x-y \end{bmatrix}, T(x,y,z) = \begin{bmatrix} x+y+z \\ y-z \end{bmatrix}$$

(c) $T \circ S$

$S: \mathbb{R}^2 \to \mathbb{R}^2 \quad T: \mathbb{R}^3 \to \mathbb{R}^2$

$T \circ S$ is not defined.

(d) $T \circ R$

$R: \mathbb{R}^2 \to \mathbb{R}^3 \quad T: \mathbb{R}^3 \to \mathbb{R}^2$, $T \circ R$ is well-defined.

$$[T \circ R] = [T][R]$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 \\ -1 & -1 \end{bmatrix}$$