Math 9 F19 Quiz 8 **Name:** Solution

(1) Find the scalar equation of the tangent plane to the surface

$$x^2 + y + z = z^2 \ln(xy)$$

at the point $(1, 1, -2)$.

Let $F(x,y,z) = x^2 + y + z - z^2 \ln(xy)$

$\nabla F(x,y,z) = \left\langle 2x - \dfrac{z^2 y}{xy},\ 1 - \dfrac{z^2 x}{xy},\ 1 - 2z\ln(xy) \right\rangle$

$\nabla F(1,1,-2) = \left\langle 2 - \dfrac{4\cdot 1}{1},\ 1 - \dfrac{4\cdot 1}{1},\ 1 - 2\cdot(-2)\cdot \ln 1 \right\rangle$

$= \langle -2, -3, 1 \rangle$

Tangent plane is

$-2(x-1) - 3(y-1) + (z+2) = 0$

(2) Let $F(x, y, z) = \begin{bmatrix} x^2 - 2z^2 \\ x^2 + y^3 \end{bmatrix}$ and $G(u, v) = \begin{bmatrix} u^3 - v \\ uv \end{bmatrix}$. Use the chain rule to find the derivative matrix $[(G \circ F)'(2, -1, 2)]$.

$$F(2,-1,2) = \begin{bmatrix} 4 - 2\cdot 4 \\ 4 + (-1)^3 \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$$

$$[G'(F(2,-1,2))] = G'(-4,3) = \begin{bmatrix} 3u^2 & -1 \\ v & u \end{bmatrix} \Bigg|_{(u,v)=(-4,3)}$$

$$= \begin{bmatrix} 48 & -1 \\ 3 & -4 \end{bmatrix}$$

$$[F'(2,-1,2)] = \begin{bmatrix} 2x & 0 & -4z \\ 2x & 3y^2 & 0 \end{bmatrix} \Bigg|_{(x,y,z)=(2,-1,2)}$$

$$= \begin{bmatrix} 4 & 0 & -8 \\ 4 & 3 & 0 \end{bmatrix}$$

Use the chain rule to obtain

$$[(G \circ F)'(2,-1,2)] = [G'(F(2,-1,2))]\,[F'(2,-1,2)]$$

$$= \begin{bmatrix} 48 & -1 \\ 3 & -4 \end{bmatrix} \begin{bmatrix} 4 & 0 & -8 \\ 4 & 3 & 0 \end{bmatrix} = \boxed{\begin{bmatrix} 188 & -3 & -384 \\ -4 & -12 & -24 \end{bmatrix}}$$