Math 9 F19 Quiz 9 **Name:** Solution

(1) Let $f(x, y) = x^2 - xy - 2y$.

(a) Classify all critical points of $f(x, y)$ (local min, local max, or saddle).

$\nabla f = \langle 2x - y, -y - 2 \rangle$

To find the critical points, solve

$2x - y = 0$

$-y - 2 = 0$

Since $y = -2$, $2x + 2 = 0$ and $x = -1$.

$(-1, -2)$ is a critical point.

$f_{xx} = 2$

$f_{xy} = -1$

$f_{yy} = -1$

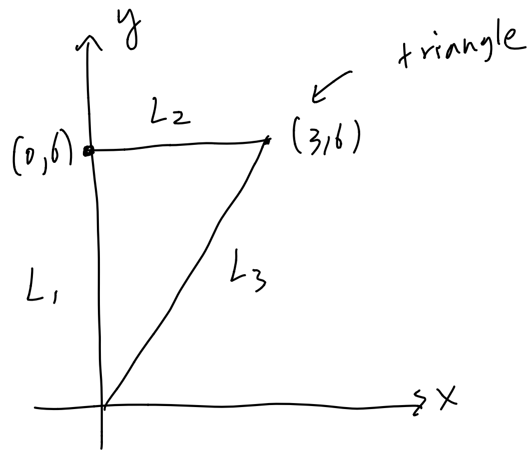
$D = 2 \cdot (-1) - (-1)^2$

$= -2 - 1 = -3 < 0$

Hence $(-2, -4)$ is a saddle.

$$f(x, y) = x^2 - xy - 2y$$

(b) Find the absolute minimum and absolute maximum of $f(x, y)$ on the triangular region with vertices $(0, 0)$, $(0, 6)$, and $(3, 6)$.



Since the critical point $(-1,-2)$ is not in the domain. We do not have to find the value at $(-1,-2)$.

L_1 : $(0,t)$ $\quad 0 \le t \le 6$. (The computation is more involved if you use $(0,6t)$ $0 \le t \le 1$.)

$f(0,t) = -2t$ does not have a critical point. endpts: $\boxed{f(0,6) = -12}$ $\boxed{f(0,0) = 0}$

L_2 : $(t,6)$, $\quad 0 \le t \le 3$

$f(t,6) = t^2 - 6t - 12$

critical pt: $2t - 6 = 0 \Rightarrow t = 3$, coincides with endpoint.

endpts: $\boxed{f(0,6) = -12}$, $\boxed{f(3,6) = -21}$ $\rightarrow$ Min

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L_3 : $(t, 2t)$, $\quad 0 \le t \le 3$

endpts: $\boxed{f(0,0) = 0}$ $\rightarrow$ Max

$\boxed{f(3,6) = -21}$

$f(t,2t) = t^2 - 2t^2 - 4t$

$= -t^2 - 4t$ $\quad$ critical pt: $-2t - 4 = 0$ $t = -2 \Rightarrow$ not in the interval.