MATHLAND MYSTERIES, SPRING 2025

Solve as many Mathland Mysteries as you can, and send them to us in a PDF along with your application. You may type or handwrite your solutions. Please write your complete solutions and not just the answers. Feel free to share your partial solutions or ideas, if you can't fully figure out a problem. You certainly don't need to solve all mysteries to be admitted to Mathland!

You may not give or receive help with the Mathland Mysteries. In particular, you may not use any books, notes, or online resources, or any computing devices (although, you may find that calculators are quite useless anyway with this type of questions).

(1) Anna and Corey are going to share a pie, and Anna cuts it into an even number of (not necessarily equal-sized) slices. Starting with Corey, they alternate taking slices, with the requirement that the missing slices must form a contiguous wedge. Is it possible for Anna to cut the pie in a way that prevents Corey from taking at least half of the pie?

(2) Anna and Corey, against their better judgement, finish the entire pie before dinner. Still, they call three more friends, and head to a restaurant. The five of them each order a different entree. When the food is ready, the waiter forgets whose is whose, and places the plates randomly on the table, one plate in front of each person. What is the probability that no one receives what they ordered?

(3) Xiang wrote the numbers from 10 to 1235 in chalk on the sidewalk.
   (a) What is the sum of all the numbers written on the sidewalk?
   (b) How many digits did Xiang write down?
   (c) What is the sum of all these digits?

(4) Xiang still has some chalk left, and challenges you to a shading game.
   (a) Xiang draws an 8×8 grid of squares and hands you the chalk. Can you shade some of the squares, so in each column there are exactly 4 shaded squares, and no two rows have the same number of shaded squares?
   (b) Xiang ups the challenge and draws a 30×30 grid. Can you shade some of the squares, so in each column there are exactly 15 shaded squares, and no two rows have the same number of shaded squares?

(5) The party gets bigger! Anna, Corey, and their three friends run into Xiang and you on their way back from the restaurant. A few more neighbors join in to admire the beautiful sidewalk art. You observe that there are now 12 of you on the sidewalk, and everyone's age is a two-digit number. One of the neighbors exclaims, "So there must be two of us whose age difference is divisible by 11!" Can you explain why?

   Your neighbor sends you home with two more mysteries to solve:

(6) Show that there is a positive number divisible by 2024 whose digits are only 0s and 1s.

(7) Is the number $222^{555} + 555^{222}$ divisible by 7?