MATH 22: HW1 - Solutions

Due Sep 21

Written homework is intended to help students develop their communication and exposition skills through complete write-ups. While correctness of the solution is, of course, necessary, much of the grade for the problem is dependent on clear and appropriate exposition. Exposition shall be appropriate for the type and level of the problem. One principle we use is that exposition should be detailed around the main aspects of the problem, but terse exposition is appropriate for subsidiary parts of a problem.

1. problem 1.1.1

   Solution:

   (a) Line

   (b) Plane

   (c) all of $\mathbb{R}^2$

2. problem 1.1.2

   Solution: [Plot]

3. problem 1.1.3

   Solution: $v = (3,3)$ and $w = (2,-2)$

4. problem 1.1.11

   Solution:

   other corners: $(1,1,1)$, $(0,1,1)$, $(1,0,1)$, $(1,1,0)$

   center of the cube: $(\frac{1}{2},\frac{1}{2},\frac{1}{2})$

   center points of six faces: $(0,\frac{1}{2},\frac{1}{2})$, $(\frac{1}{2},0,\frac{1}{2})$, $(\frac{1}{2},\frac{1}{2},0)$, $(1,\frac{1}{2},\frac{1}{2})$, $(\frac{1}{2},1,\frac{1}{2})$, $(\frac{1}{2},\frac{1}{2},1)$

5. problem 1.1.26

Solution: $c + 3d = 14,\ 2c + d = 8 \Rightarrow c = 2,\ d = 4.$

6. problem 1.2.1

Solution: $u \cdot v = 1.4,\ u \cdot w = 0,\ u \cdot (v + w) = 1.4,\ w \cdot v = 48$

7. problem 1.2.2

Solution:

$\|u\| = 1,\ \|v\| = 5,\ \|w\| = 10.$

$$|u \cdot v| = 1.4 \leq \|u\|\|v\| = 5$$

$$|v \cdot w| = 48 \leq \|v\|\|w\| = 50$$

8. problem 1.2.8

Solution:

(a) False. $u = (1, 0, 0), v = (0, 1, 0), w = (0, 0, 1).$

(b) True. $u \cdot (v + 2w) = u \cdot v + 2u \cdot w = 0 + 0 = 0.$

(c) True. $\|u - v\|^2 = \|u\|^2 + \| - v\|^2 - 2u \cdot v = 1 + 1 - 0 = 2.$

9. problem 1.2.13

Solution: $v = (1, 0, -1),\ w = (0, 1, 0)$

10. problem 1.2.14

Solution: $u = (1, -1, 1, -1),\ v = (1, 1, -1, -1), w = (1, -1, -1, 1)$

11. problem 1.3.1

Solution:

$$2s_1 + 3s_2 + 4s_3 = \begin{bmatrix} 2 \\ 5 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 9 \end{bmatrix}$$

12. problem 1.3.2 (giving a formula for the first n odd numbers is optional).

Solution:

First equation: $y = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

Second equation: $y = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$.

[problem numbering is based on the textbook 'Introduction to Linear Algebra by Gilbert Strang, 4th edition]