MATH 22: HW 4 - Solutions

[Due Oct 12]

Written homework is intended to help students develop their communication and exposition skills through complete write-ups. While correctness of the solution is, of course, necessary, much of the grade for the problem is dependent on clear and appropriate exposition. Exposition shall be appropriate for the type and level of the problem. One principle we use is that exposition should be detailed around the main aspects of the problem, but terse exposition is appropriate for subsidiary parts of a problem.

1. problem 3.1.4

Solution:

$$\vec{0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\frac{1}{2}A = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$

$$-A = \begin{bmatrix} -2 & 2 \\ -2 & 2 \end{bmatrix}$$

The smallest subspace containing A contains any matrix of the form

$$\begin{bmatrix} a & -a \\ a & -a \end{bmatrix}$$

where a is an arbitrary real number.

2. problem 3.1.7

Solution:
From the rules listed at the beginning of problem set (page 127), the rule (7) fails in this example. Let us explain this in more depth. Every vector space comes with a recipe on how to multiply a number $c \in \mathbb{R}$ with a vector v to get a new vector cv. For instance, for the standard vector space $\mathbb{R}^2$ we define cv to be a the vector v scaled by a factor of c. In the case of functions, if we define cf by $(cf)(x) = f(cx)$ then

$$(c_1 + c_2)\, f(x) = f(c_1 x + c_2 x)$$
$$c_1 f(x) + c_2 f(x) = f(c_1 x) + f(c_2 x)$$

This means that we need to have $f(c_1 x + c_2 x) = f(c_1 x) + f(c_2 x)$ but this is not true for a general function. For example, if $f(x) = x^2$ and $c_1 = c_2 = 1$ then $(x + x)^2 \neq x^2 + x^2$.

3. problem 3.1.10

Solution:
(a) Yes

(b) No

(c) No

(d) Yes

(e) Yes

(f) No

4. problem 3.1.16

Solution: A plane or the entire 3D space.

5. problem 3.1.18

Solution:
(a) True

(b) True

(c) False

6. problem 3.1.19

Solution:
The column space of A is

$$\begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix} \quad \textit{where } a \in \mathbb{R}$$

The column space of B is

$$\begin{bmatrix} a \\ b \\ 0 \end{bmatrix} \quad \textit{where } a, b \in \mathbb{R}$$

The column space of C is

$$\begin{bmatrix} a \\ 2a \\ 0 \end{bmatrix} \quad \textit{where } a \in \mathbb{R}$$