NAME: ______________________________

SECTION: (Latifi 10:10) ☐ (Mintz 11:30) ☐

Math 22

Fall 2023

Final Exam

November 17th, 2023

Instructions:

1. Please read all the instructions, and read and sign the statement at the bottom of this page.

2. Print your name *legibly* at the top of this page, and indicate your section by checking the appropriate box.

3. Some problems have multiple parts, do all of them. The different parts of a problem are not necessarily worth the same number of points.

4. Except on clearly indicated short answer problems, you must explain what you are doing, and show your work. You will be *graded on your work*, not just on your answer. Make it clear and legible so we can follow it.

5. It is fine to leave your answer in a form such as $\ln(.02)$ or $\sqrt{239}$ or $(385)(13^3)$. However, if an expression can be easily simplified (such as $e^{\ln(.02)}$ or $\cos(\pi)$ or $(3-2)$), you should simplify it.
Answers should *not* be rounded. If the answer is π then 3.14159 is wrong.
If the problem includes units, your answer should include units.

6. There are a few pages of scratch paper at the end of the exam. We *will not look* at these pages unless you write on a problem "Continued on page…"

7. Once finished, you will upload your exam to Gradescope. **You must assign pages to each question so that we can return your scores in a reasonable timeframe.** Once the exam is uploaded, give your physical copy to the instructors in case there is a problem with the upload. If you finish early, you should find the attending instructor before uploading so as not to distract others.

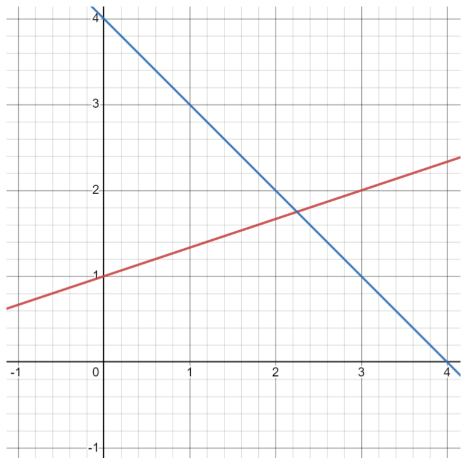
8. **Honor Code:** This exam is closed book. You may not consult notes, calculators (except as explicitly permitted for accessibility), phones, or any other external resource. Please turn off your devices until you have finished the exam.
It is a violation of the honor code to give or receive help on this exam.

STATEMENT: I have read these instructions, and I understand how the honor code applies to this exam.

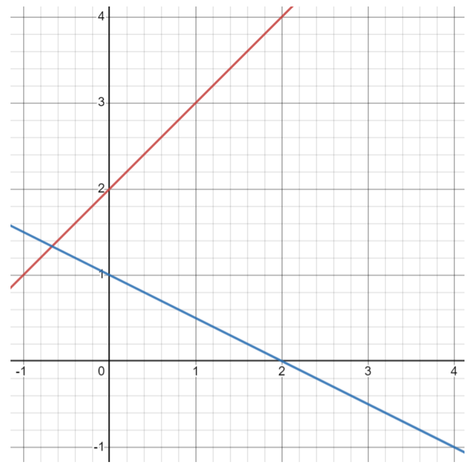
SIGNATURE: ______________________________

1. (10 points) Which of the following graph shows the following matrix equation in row picture

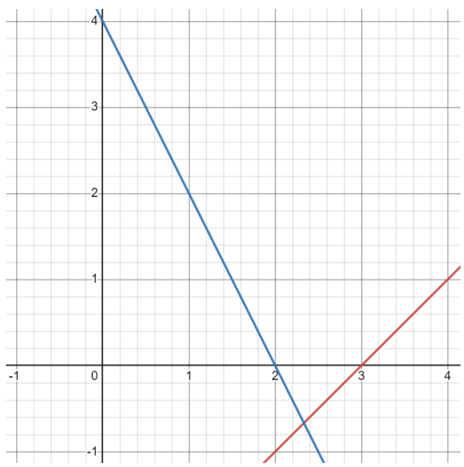
$$\begin{bmatrix} -1 & 1 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$$



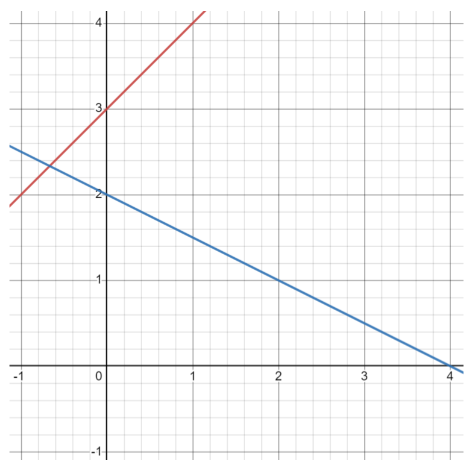
(a)



(b)



(c)



(d)

2. (10 points) Solve the system $A\vec{x} = \vec{b}$, where

$$A = LU = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad \vec{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

One approach is to solve $L\vec{y} = \vec{b}$, then solve $U\vec{x} = \vec{y}$ so that $A\vec{x} = LU\vec{x} = L\vec{y} = \vec{b}$.
Note each matrix is triangular, so you only have to use back-substitution.

3. (10 points) Let $P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and $A = \begin{bmatrix} 1 & 5 & 9 & 13 \\ 2 & 6 & 10 & 14 \\ 3 & 7 & 11 & 15 \\ 4 & 8 & 12 & 16 \end{bmatrix}$.

1. What happens to A when we take the product AP? Explain in words what happens to its rows and/or columns. You do not necessarily need to find this matrix.
2. What would happen if we left-multiplied by P instead? Explain in words what happens to its rows and/or columns. You do not necessarily need to find this matrix.

4. (10 points) Which of the four fundamental subspaces of the follow matrix have dimension two? You don't need to find all of these explicitly.

$$A = \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & 2 & -1 \\ 1 & 0 & -2 & 4 \end{bmatrix}$$

5. (10 points) The equation $A\vec{x} = \vec{b}$ has the complete solution

$$\left\{ \begin{bmatrix} 0 \\ -1 \\ 4 \\ 1 \\ 0 \end{bmatrix} + a \begin{bmatrix} 2 \\ 3 \\ 0 \\ 1 \\ 2 \end{bmatrix} + b \begin{bmatrix} 0 \\ -1 \\ 2 \\ 0 \\ 0 \end{bmatrix} + c \begin{bmatrix} -1 \\ 1 \\ 0 \\ -4 \\ -2 \end{bmatrix}, \text{ for } a, b, c, \text{ in } \mathbb{R} \right\}$$

1. Find three particular solutions to this equation.
2. What are the number of rows and columns of A? Note we may not be able to determine one, or both.

6. (5 points) Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \\ 1 & 2 & 3 \\ 2 & 1 & 2 \end{bmatrix}$$

Can $S = C(A)$ and $T = C(B)$ be perpendicular complements? Note the columns of A are linearly independent, as are the columns of B.

7. (10 points) Use the properties of the determinant to show a triangular matrix with a zero on it's diagonal cannot be invertible.

8. (10 points) If $\vec{v}$ is an eigenvector of A with eigenvalue λ, is $\vec{v}$ an eigenvector of $A^2 + I$? If so, what is the eigenvalue, and if not, find a matrix A and eigenvector $\vec{v}$ where this is not the case.

9. Find the determinant of the following matrix. Provide a name for the formula you are using.

$$A = \begin{bmatrix} 2 & 0 & 5 \\ 0 & 1 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

10. Find all solutions for the system

$$\begin{cases} x_1 + 2x_2 + x_3 = 9 \\ -x_1 + x_2 - x_3 - x_4 = 0 \end{cases}$$

11. Determine the number α such that 3 is an eigenvalue of

$$A = \begin{bmatrix} 1 & \alpha \\ 2 & 3 \end{bmatrix}$$

Extra room for scratch work. We will NOT look at this page, unless you write on another page "continued on page..."

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