NAME: ______________________________

SECTION: (Latifi 10:10) ☐ (Mintz 11:30) ☐

Math 22: Linear Algebra With Applications

Fall 2023

Practice for Midterm 1

Instructions:

1. Please read all the instructions, and read and sign the statement at the bottom of this page.

2. Print your name *legibly* at the top of this page, and indicate your section by checking the appropriate box.

3. Some questions may have multiple parts. Do all of them. The different parts of a problem are not necessarily worth the same number of points.

4. Except on clearly indicated short answer problems, you must explain what you are doing, and show your work. You will be *graded on your work*, not just on your answer. Make it clear and legible so we can follow it.

5. It is fine to leave your answer in a form such as $\ln(.02)$ or $\sqrt{239}$ or $(385)(13^3)$. However, if an expression can be easily simplified (such as $e^{\ln(.02)}$ or $\cos(\pi)$ or $(3-2)$), you should simplify it.
Answers should *not* be rounded. If the answer is π then 3.14159 is wrong.
If the problem includes units, your answer should include units.

6. There are a few pages of scratch paper at the end of the exam. We *will not look* at these pages unless you write on a problem "Continued on page…"

7. Once finished, you will upload your exam to Gradescope. **You must assign pages to each question so that we can return your scores in a reasonable timeframe.** Once the exam is uploaded, give your physical copy to the instructors in case there is a problem with the upload. If you finish early, you should find the attending instructor before uploading so as not to distract others.

8. **Honor Code:** This exam is closed book. You may not consult notes, calculators (except as explicitly permitted for accessibility), phones, or any other external resource. Please turn off your devices until you have finished the exam.
It is a violation of the honor code to give or receive help on this exam.

STATEMENT: I have read these instructions, and I understand how the honor code applies to this exam.

SIGNATURE: ______________________________

1. (10 points) Find the sum of each pair of the below vectors: $\vec{u}+\vec{v}$, $\vec{u}+\vec{w}$, and $\vec{v}+\vec{w}$. Which sum gives the longest vector?

$$\vec{u} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \qquad \vec{v} = \begin{bmatrix} 2 \\ -4 \\ -5 \end{bmatrix}, \qquad \vec{v} = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix},$$

2. (10 points) Are the vectors $\vec{v}$ and $r\vec{v}$ ever perpendicular, that is, can you find a real number r and vector $\vec{v}$ where these are perpendicular (other than the trivial solution $\vec{v} = \vec{0}$ or $r = 0$)?

3. (10 points) Fill in the missing entries a and b of the matrix so that the system has no solution. Explain why this is the case, either with the row or column view. A graph may be helpful to explain yourself.

$$\begin{bmatrix} 1 & a \\ -2 & b \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

4. (10 points) Find inverse of the following matrix using Gauss Jordan method

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

5. (10 points) If all linear combinations of columns of a 3 by 3 matrix A span a **plane** in 3D space, can we always solve $A\vec{x} = \vec{b}$? If no, under what condition on $\vec{b}$ we can solve this system?

6. (10 points) Solve $A\vec{x} + B\vec{x} = \vec{b}$ for the given matrices

$$A = \begin{bmatrix} 0 & -2 & 1 \\ 5 & 2 & 3 \\ 0 & 0 & -1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 2 & 0 \\ -3 & 1 & 8 \\ 0 & -1 & 5 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} 2 \\ 10 \\ 5 \end{bmatrix}$$

Hint: use distributivity to rewrite this sum as a single product.

7. (10 points) If $\vec{u}$ is perpendicular to $\vec{v}$ and $\vec{v}$ is perpendicular to $\vec{w}$, then must $\vec{u}$ be perpendicular to $\vec{w}$?

8. Let

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \qquad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

   (a) Find A^2 and A^3.
   (b) Evaluate the expression $I + A + A^2 + A^3 + A^4 + \dots$.

9. Suppose that a "Regular" can of mixed nuts comes with a cashews/peanuts ratio of 1:3 (1 lb cashews for 3 lb peanuts) while a can of "Deluxe" has cashews/peanuts ratio of 1:1 (1 lb cashews for 1 lb peanuts). Can you obtain a mixture of 2 lb cashews and 3 lb peanuts by combining the two mixtures in appropriate amounts? (Hint: let x_1 and x_2 be the amount of Regular and Deluxe cans, respectively, and solve for x_1 and x_2).

10. Find elimination matrices E_{21} and E_{32} that turn the following system into an upper diagonal form. Use E_1 and E_2 matrices to construct a factorization of the form $A = LU$ (in terms of lower and upper triangular matrices).

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -3 & 1 & 0 \\ 0 & 6 & 4 \end{bmatrix}$$

Extra room for scratch work. We will NOT look at this page, unless you write on another page "continued on page..."

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