Homework 2 - Problem Set

(Numbered according to 9th edition)
Problem 1: Section 2.1 #13

Problem 2: Section 2.1 #14

Problem 3: Section 2.2 #4

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Problem 10: Section 2.6 #3

Section 2.1

#13

Using the integrating factor e^{-t} we have

$$e^{-t}y' - e^{-t}y = 2te^t$$

since $e^{-t}y' = (e^{-}ty)' + e^{-t}y$ we get

$$(e^{-}ty)' = 2te^t$$

and hence

$$e^{-t}y = \int_{t_0}^{t} 2se^s\, ds + c = 2e^t(t-1) + c$$

the initial condition $y(0) = 1$ implies $c = 3$ and

$$y(t) = 2e^{2t}(t-1) + 3e^t$$

#14

$$y' + 2y = te^{-2t}$$

Integrating factor: $\mu(t) = e^{2t}$

$$\Rightarrow \left(e^{2t}y\right)' = t$$
$$\Rightarrow e^{2t}y = \frac{t^2}{2} + c$$
$$\Rightarrow y = \frac{t^2}{2}e^{-2t} + ce^{-2t}$$

Boundary condition $y(1) = 0 \Rightarrow c = \frac{-1}{2}$

$$\Rightarrow \text{ solution:} \quad y(t) = \frac{e^{-2t}}{2}\left(t^2 - 1\right)$$

Section 2.2

#4

$$y' = \frac{3x^2 - 1}{3 + 2y}$$

Separation of variables:

$$(3 + 2y)dy + \left(1 - 3x^2\right)dx = 0$$
$$\Rightarrow \begin{cases} H_1'(x) = 1 - 3x^2 \\ H_2'(y) = 3 + 2y \end{cases} \Rightarrow \begin{cases} H_1(x) = x - x^3 \\ H_2(y) = 3y + y^2 \end{cases}$$

$\Rightarrow$ solution:

$$x - x^3 + 3y + y^2 = c$$

#8

To solve

$$\frac{dy}{dx} = \frac{x^2}{1 + y^2}$$

we separate it as

$$-(x^2)\,dx + (1 + y^2)\,dy = 0$$

and observe that if $H_1'(x) = -x^2$ and $H_2'(y) = 1 + y^2$ then

$$H_1(x) + H_2(y) = -\frac{x^3}{3} + y + \frac{y^3}{3} + c_0$$

and the solutions are given by

$$-x^3 + 3y + y^3 = c$$

#32

We can rewrite this homogeneous differential equation as

$$\frac{dy}{dx} = \frac{1 + 3(y/x)^2}{2(y/x)}$$

using $v = y/x$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ we have

$$x\frac{dv}{dx} = \frac{1+v^2}{2v}$$

using separation of variables and if $H_1'(x) = -1/x$ and $H_2'(v) = 2v/(1+v^2)$ then

$$H_1(x) + H_2(v) = \ln(1+v^2) - ln(x)$$

hence the solution is given by $\ln((1+v^2)/x) = c_0$ which is equivalent to

$$x^2 + y^2 - cx^3 = 0$$

Section 2.3

#3

Solution: $50e^{-0.2}(1-e^{-0.2})$lb $(7.42lb)$

Section 2.4

#25

To verify that $y_1 + y_2$ is a solution if (ii) we write

$$(y_1+y_2)' + p(y_1+y_2) = (y_1' + py_1) + (y_2' + py_2) = 0 + g(t)$$

Section 2.5

#2

$$\frac{dy}{dt} = ay + by^2 = y(a+by)$$

$y(t) = 0 \rightarrow$ stable equilibrium solution
$y(t) = \frac{-a}{b} \rightarrow$ unstable equilibrium solution

#3

$$\frac{dy}{dt} = y(y-1)(y-2)$$

stable equilibrium solutions: $y(t) = 1$
unstable equilibrium solution:

$$y(t) = 0$$
$$y(t) = 2$$

Section 2.6

#3

Exact equation

$$\left(3x^2 - 2xy + 2\right) dx + \left(6y^2 - x^2 + 3\right) dy = 0$$
$$\Rightarrow \begin{cases} F(x,y) = x^3 - x^2y + 2x + h(y) \\ F(x,y) = 2y^3 - yx^2 + 3y + k(x) \end{cases}$$
$$\Rightarrow h(y) = 2y^3 + 3y$$

$\Rightarrow$ Solution: $F(x,y) = c$

$$x^3 - x^2y + 2x + 2y^3 + 3y = c$$