Homework 3 - Problem Set

(Numbered according to 9th edition)
Problem 1: Section 3.1 #10

Problem 2: Section 3.1 #15

Problem 3: Section 3.1 #18

Problem 4: Section 3.3 #12

Problem 5: Section 3.3 #40 (read #34 first)

Problem 6: Section 3.3 #46* [optional] (read #43 first)

3.1.10

$$y'' + 4y' + 3y = 0$$
$$y(0) = 2, y'(0) = -1$$

characteristic equation:

$$r^2 + 4r + 3 = 0 \Rightarrow r_1 = -3, r_2 = -1$$

general solution:

$$y(t) = c_1 e^{-t} + c_2 e^{-3t}$$
$$\begin{cases} y(0) = 2 \\ y'(0) = -1 \end{cases} \Rightarrow \begin{cases} c_1 + c_2 = 2 \\ -c_1 - 3c_2 = -1 \end{cases}$$
$$\Rightarrow \begin{cases} c_1 = \frac{5}{2} \\ c_2 = -\frac{1}{2} \end{cases}$$

solution $y(t) = \frac{5}{2}e^{-t} - \frac{1}{2}e^{-3t}$

3.1.15

$$y'' + 8y' - 9y = 0$$
$$y(1) = 1, \quad y'(1) = 0$$

characteristic equation:

$$r^2 + 8r - 9 = \rightarrow r = \frac{-8 \pm \sqrt{100}}{2}$$
$$\Rightarrow r_1 = 1, r_2 = -9$$

general solution

$$y(t) = c_1 e^{t} + c_2 e^{-9t}$$

given boundary conditions we get

$$y(t) = \frac{9}{10}e^{t-1} + \frac{1}{10}e^{-9(t-1)}$$

with $y \to \infty$ as $t \to \infty$.

3.1.18

$$y(t) = c_1 e^{-t/2} + c_2 e^{-2t}$$
$$\Rightarrow r_1 = \frac{-1}{2}, r_2 = -2$$
$$\Rightarrow \quad \text{characteristic } eq: \quad \left(r + \frac{1}{2}\right)(r + 2) = 0$$
$$r^2 + \frac{5r}{2} + 1 = 0$$
$$2r^2 + 5r + 2 = 0$$

equation:

$$2y'' + 5y' + 2y = 0$$

3.3.12

$$4y'' + 9y = 0$$

characteristic equation:

$$4r^2 + 9 = 0 \to r = \sqrt{\frac{-9}{4}}$$
$$\Rightarrow r_1 = \frac{3i}{2}, r_2 = -\frac{3i}{2}$$

General solution in complex form:

$$y(t) = c_1 e^{\frac{3}{-}it} + c_2 e^{-\frac{3}{2}it}$$

General solution in real form:

$$y(t) = c_1 \cos\left(\frac{3}{2}t\right) + c_2 \sin\left(\frac{3}{2}t\right)$$

3.3.40

$$t^2\frac{d^2y}{dt^2} - t\frac{dy}{dt} + 5y = 0$$

change of variable $x = \ln(t)$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 0$$

characteristic eq: $r^2 - 2r + 5 = 0$

$$\Rightarrow r = \frac{2 \pm \sqrt{4 - (4)(5)}}{2}$$
$$\Rightarrow r_1 = 1 + 2i, r_2 = 1 - 2i$$
$$\Rightarrow y(x) = c_1 e^x \cos(2x) + c_2 e^x \sin(2x)$$
$$\Rightarrow y(t) = c_1 t \cos(2\ln t) + c_2 t \sin(2\ln t)$$