

Homework 8 - Problem Set

(Numbered according to 9th edition)

Problem 1: Section 9.1 #20. Consider the linear system

$$dx/dt = a_{11}x + a_{12}y, \quad dy/dt = a_{21}x + a_{22}y$$

where a_{11}, \dots, a_{22} are real constants. Let $p = a_{11} + a_{22}$, $q = a_{11}a_{22} - a_{12}a_{21}$, and $\Delta = p^2 - 4q$. Observe that p and q are the trace and determinant, respectively, of the coefficient matrix of the given system. Show that the critical point $(0, 0)$ is a

- (a) Node if $q > 0$ and $\Delta \geq 0$.
- (b) Saddle point if $q < 0$.
- (c) Spiral point if $p \neq 0$ and $\Delta < 0$
- (d) Center if $p = 0$ and $q > 0$.

Hint: These conclusions can be obtained by studying the eigenvalues r_1 and r_2 . It may also be helpful to establish and then to use, the relations $r_1 r_2 = q$ and $r_1 + r_2 = p$.

Problem 2: Section 9.1 #21. Continuing Problem 20, show that the critical point $(0, 0)$ is

- (a) Asymptotically stable if $q > 0$ and $p < 0$;
- (b) Stable if $q > 0$ and $p = 0$;
- (c) Unstable if $q < 0$ or $p > 0$.
- (d) Center if $p = 0$ and $q > 0$.

(the results of Problems 20 and 21 are summarized visually in Figure 9.1.9. page 497)