1. Find the solution for

$$y' + (2/t)y = (\cos t)/t^2, \quad y(\pi) = 0, \quad t > 0$$

2. Find the solution for

$$2y' - y = e^{t/3}, \quad y(0) = a$$

3. Find the solution for

$$y' + y^2 \sin x = 0$$

4. Find the solution for

$$y'' + 3y' = 0, \quad y(0) = -2, \quad y'(0) = 3$$

5. Find a differential equation whose general solution is $y = c_1 e^{-t/2} + c_2 e^{-2t}$.

6. Find the solution for

$$y'' - 2y' + 5y = 0, \quad y(\pi/2) = 0, \quad y'(\pi/2) = 2$$

7. Using Euler's formula, show that

$$\cos t = \left(e^{it} + e^{-it}\right)/2, \quad \sin t = \left(e^{it} - e^{-it}\right)/2i$$

8. Find the solution for

$$y'' - 2y' + 10y = 0$$

9. Find the solution for

$$y'' - 3y' - 4y = 2\sin t$$

10. Find the solution for

$$y'' - 2y' - 3y = -3te^{-t}$$

11. Find the solution for

$$y'' + 2y' + y = 3e^{-t}$$

12. Solve the given initial value problem. Describe the behavior of the solution as $t \to \infty$.

$$\mathbf{x}' = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix}\mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

13. Solve the given initial value problem. Describe the behavior of the solution as $t \to \infty$.

$$\mathbf{x}' = \begin{pmatrix} -2 & 1 \\ -5 & 4 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

14. Given the eigenvalues and eigenvectors sketch a phase portrait of the system (Identify the type of the critical point)

$$r_1 = -1, \quad \xi^{(1)} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}; \quad r_2 = -2, \quad \xi^{(2)} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

15. Find the solution for

$$\mathbf{x}' = \begin{pmatrix} 1 & -5 \\ 1 & -3 \end{pmatrix}\mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

16. Solve the initial value problem

$$\mathbf{x}' = \begin{pmatrix} -1 & -4 \\ 1 & -1 \end{pmatrix}\mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

using the fundamental matrix $\mathbf{\Phi}(t)$ (first find a fundamental matrix).

17. Show that if $\mathbf{A}$ is a diagonal 2x2 matrix with diagonal elements a_1, a_2, then $\exp(\mathbf{A}t)$ is also a diagonal matrix with diagonal elements $\exp(a_1 t), \exp(a_2 t)$.

18. Find the general solution of

$$\mathbf{x}' = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 0 \\ \cos t \end{pmatrix}, \quad 0 < t < \pi$$

19. Given

$$dx/dt = (2+x)(y-x), \quad dy/dt = (4-x)(y+x)$$

(a) Determine all critical points of the given system of equations.
(b) Find the corresponding linear system near each critical point.
(c) Find the eigenvalues of each linear system. What conclusions can you then draw about the nonlinear system?
(d) Draw a phase portrait of the nonlinear system using an online tool and compare it with your findings in a-c.

20. Given

$$dx/dt = x + x^2 + y^2, \quad dy/dt = y - xy$$

(a) Determine all critical points of the given system of equations.
(b) Find the corresponding linear system near each critical point.
(c) Find the eigenvalues of each linear system. What conclusions can you then draw about the nonlinear system?
(d) Draw a phase portrait of the nonlinear system using an online tool and compare it with your findings in a-c.

21. Use the Laplace transform to solve the given initial value problem

$$y'' - y' - 6y = 0; \quad y(0) = 1, \quad y'(0) = -1$$