Grading

Problems will be graded for accuracy and clarity of both mathematics and writing.

Problems

1. **Classification of Differential Equations**
   In each of the below problems, determine the order of the given differential equation and state whether the equation is linear or nonlinear. Justify your classification.

   (a) $t^2 \frac{d^2 y}{dt^2} + t \frac{dy}{dt} + 2y = \sin t$

   (b) $(1 + y^2) \frac{d^2 y}{dt^2} + t \frac{dy}{dt} + y = e^t$

   (c) $\frac{d^4 y}{dt^4} + \frac{d^3 y}{dt^3} + \frac{d^2 y}{dt^2} + \frac{dy}{dt} + y = 1$

   (d) $\frac{d^2 y}{dt^2} + \sin(t + y) = \sin t$
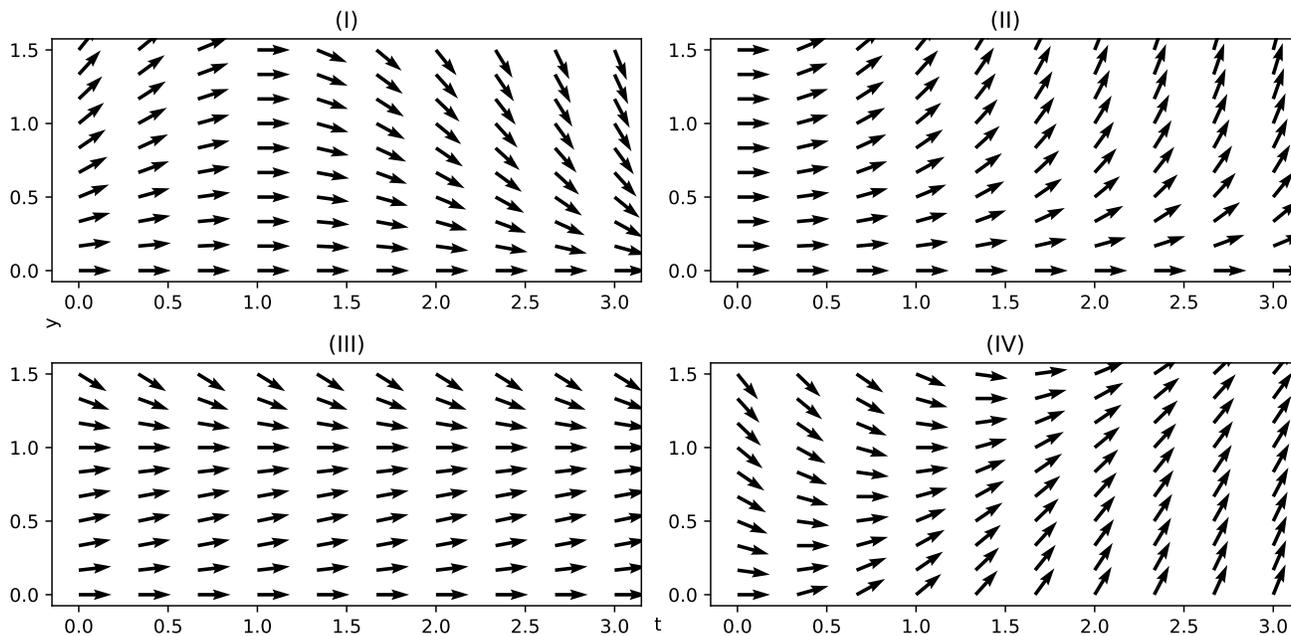
2. Consider the following ODEs

    (a) $\frac{d}{dt}y = y(1-y)$

    (b) $\frac{d}{dt}y = yt$

    (c) $\frac{d}{dt}y = t - y$

    (d) $\frac{d}{dt}y = y + ty$

    Identify which direction field corresponds to each ODE

3. Find the solution to the IVP

$$\frac{d}{dt}y = y + t^2, \quad y(0) = 1$$

and describe the long-term (large t behavior of the solution)

4. Solve the initial value problem

$$\frac{d}{dt}y = \frac{y^2}{t}, \quad y(1) = 1$$