

**Grading**

Problems will be graded for accuracy and clarity of both mathematics and writing.

**Problems**

1. Find a differential equation that has the solution

$$y(t) = e^{-4t} + 5e^{-2t}.$$

What are some initial values that are satisfied by this equation.

2. Consider a spring-mass system governed by the equation

$$x'' = -4x' - 5x$$

with initial state  $x(0) = 0$  and  $x'(0) = 1$ .

(a) Find an expression for the motion of the spring-mass as a function of time  $t$ .

(b) What is the largest distance of the spring-mass to the origin for  $0 \leq t < \infty$ ?

(c) Describe the long-term behavior of mass-spring as  $t \rightarrow \infty$ .

3. Find the general solution to

$$y'' + y = e^{2t}. \quad (1)$$

4. Find the general solution to

$$9y'' + 6y' + y = 0 \quad (2)$$

5. Write down the solution to the following IVP problem.

$$\frac{d}{dt}\mathbf{x}(t) = \begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix} \mathbf{x}(t)$$
$$\mathbf{x}(0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$