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Misleading Averages

Suggested grade levels: 11 and up due to subject matter and reading levels.

Possible subject areas: Social studies

Math skills: Arithmetic, percents and averages.

Overview: The average (or mean) of a list of numbers is their sum divided by the number of numbers in the set. What about an average *rate*, such as an average percent? For example how does one define the average rate of taxation of a group of people? We will see that averages can be misleading – especially if one is not careful about what an average rate means.

Student Activities: Misleading Averages

The average (or mean) of a set of numbers is their sum divided by the number of numbers in the set.

Sometimes an average or a percentage can be misleading or may require thoughtful interpretation. For example if Annie's Autos says the average price of their used cars is $9000 you would probably expect to find a lot of cars at Annie's in that price range.

However, if Annie has two autos for sale, one at $1000 and one at $17000, the average is $9000, but there aren't any cars in the $9000 price range. The average doesn't really tell us much about the numbers that produced the average.

The examples below, like the simple one above, are designed to illustrate that you need to be careful in drawing conclusions from averages of data. Among other things, it is often important to know where the data set comes from and how it is distributed.

Example I. Ms. Smith taught 3 Math classes last term. Student evaluations for her courses came out as follows.

Sec 1: 1 B 25 A
Sec 2: 4 B 22 A
Sec 3: 50 E 40 D 20 C

A (excellent) is 5 points, B = 4 points, C = 3 points, D = 2 points, and E = 1 point.

1. What is Ms. Smith's average score in class 1?

2. What is Ms. Smith's average score in class 2?

3. What is Ms. Smith's average score in class 3?

4. Instructors in the mathematics department are rated on their overall teaching evaluations. There are two ways to think about Ms. Smith's overall average evaluation:

   (a) One can propose averaging the three average scores in the classes computed above to get a composite. What would that number be?

   (b) One can aggregate all classes together as if they were one class. That means you would add up the points and divide by the total number of students in all classes. What would that number be?

5. Discuss the merits and drawbacks to the two methods.

6. Bill was trying to decide whether to enroll in one of Ms. Smith's math classes or not. What advice would you give him?

Example II. The following article was found on the web site http://www.cbpp.org/taxday98.htm and is reprinted by permission. (See endnote.)

Tax Foundation Figures Produce Misleading and Inaccurate Impressions of Middle Class Tax Burdens

by Iris J. Lav, Isaac Shapiro, and Robert Greenstein

The Tax Foundation's use of averages is severely flawed and substantially exaggerates middle-class tax bills.

In contending the average American must work until May to pay taxes, the Foundation takes what it says is the total amount paid in federal, state, and local taxes and simply divides this amount by the Foundation's estimate of the total amount of income in the nation. The result is the percentage of income the Tax Foundation pictures the average American as paying in taxes, which the Foundation then converts into the percentage of the year the average American must work to satisfy his or her tax obligations. This methodology assumes that everyone pays the same percentage of income in taxes, which of course is not correct. The wealthy pay a substantially higher percentage of income in taxes than the middle class or the poor do.

The federal personal income tax, for example, is a progressive tax. The typical middle-income family is in the 15 percent federal income tax bracket. High-income families are in brackets with marginal rates more than twice that high and pay substantially higher percentages of income in federal income tax than middle-class families do.

The problem of using averages as the Tax Foundation does is easily seen. Suppose four families with $25,000 incomes each pay $1,250 in income tax – or five percent of their

income – while one wealthy family with $500,000 in income pays $125,000 in income tax, or 25 percent of its income. These five families pay an average of 22 percent of their income in federal income taxes (total tax payments of $130,000 divided by total income of $600,000).

But the 22 percent figure is misleading. The four moderate-income families pay five percent of their income in income tax, not 22 percent. Using averages when talking about tax burdens produces skewed results; it ascribes tax rates to the average person that only taxpayers at considerably higher income levels pay.

The Tax Foundation's averaging approach is flawed in the same regard with respect to various other types of taxes. For example, the Tax Foundation assumes that middle-class families pay the same percentage of income in estate taxes as a family with a multi-million dollar income. But estate taxes are paid on only the largest one to two percent of estates; all smaller estates are exempt.

This holds true for corporate income taxes, as well, which most economists (including those at CBO) *[Editorial remark: CBO stands for "Congressional Budget Office"]* believe are primarily passed through to the owners of capital assets, and for capital gains taxes paid on the sale of stocks, bonds, and real estate. The Tax Foundation erroneously assumes that typical middle-class families pay the same percentage of income in corporate income and capital gains tax as wealthy investors and stockholders.

[Editorial remark: The following paragraph is somewhat confusing and may contain a typo.] Both Congressional Budget Office tables and Joint Committee on Taxation tables show that the average federal tax burden is higher than the federal tax burden that even the next-to-the-top income fifth pays (those between the 60th and 80th percentiles in income). These CBO and Joint Tax tables show how misleading it is to present the Tax Foundation's results as though they apply to families in the middle of the income scale.

Now that you have read the above article, try to answer the following questions:

7. Suppose seven families with $20,000 incomes each pay $1,200 in income tax. What percent of their income do these families pay?

8. Suppose one wealthy family with $800,000 in income pays $224,000 in income tax. What percent of its income does this family pay?

9. Approximately what percent do the eight families above pay on average? (As in the article above, the definition of "average tax rate" is the total tax revenue divided by the total income. It is that rate such that if all members of the group paid that rate the tax revenue generated would be the same as before.)

10. Suppose someone were to suggest that the average tax rate could be obtained by actually averaging the percentages. What is this number? Explain why this number is not the average tax rate.

For the Teacher:

Example I.

1. Ms. Smith's average score in class 1 is 4.96

2. Ms. Smith's average score in class 2 is 4.85

3. Ms. Smith's average score in class 3 is 1.73

4. Different ways of averaging:

   (a) One can propose averaging the three average scores in the classes computed above to get a composite. That number would be 3.85.

   (b) One can aggregate all classes together as if they were one class. That means you would add up the points and divide by the total number of students in all classes. That number would be 2.74.

5. Discuss the merits and drawbacks to the two methods: The average in (b) gives the ratings of each of the 162 students the same weight - as if all of them were in one class. The average in (a) ignores the differences in the number of students in each class, giving as much weight to the two small classes as the one large class. On the other hand, aggregating all three classes masks the fact that Ms. Smith's ratings were poor in the large class and good in the smaller ones.

6. Bill was trying to decide whether to enroll in one of Ms. Smith 's math classes or not. The data indicate that Ms. Smith is much better with small classes than large ones. Perhaps the best advice would be to take Ms. Smith if she is teaching a small class, but avoid her if she's teaching a big class. This is another example where an average alone does not necessarily convey a complete picture. One needs to be alert as to where the data came from and how the averages were arrived at.

Example II. Taxes:

7. The seven making $20,000 pay 6% of their income.

8. The rich family pays 28%.

9. These eight families pay an average of about 25 percent (24.72%) of their income in federal income taxes (total tax payments of $232,400 divided by total income of $940,000). This is in accord with the definition: if all eight people paid 24.72% of their income in taxes, the revenue would be the same as before. That is, 24.72% of $940,000 is $232,400.

10. Multiply 6% by 7 and then add 28% to get 70%. Divide 70% by 8 to get 8.75%. This is not the average tax rate because, by definition, the "average tax rate" is that rate such that if all members of the group paid that rate the revenue would be the same. But if everyone paid 8.75%, then the revenue would be 8.75% of $940,000, which is $82,250, not $232,400. It is simply a consequence of the definition that "average tax rate" is not the same as "the average of all the tax rates."

Remarks: A similar kind of situation arises when figuring the average speed of a car. Suppose you are on a 240-mile trip. If you drive the first 120 miles at 40 mph. and then the second 120 miles at 60 mph what is your average speed for the 240 - mile trip? (Assume no loss of time in changing speeds.)

Most students will answer 50 mph, which they get by averaging the two speeds 40 and 60. This is incorrect. At a speed of 40 mph, it takes 3 hours to go 120 miles. At a speed of 60 mph, it takes 2 hours to go 120 miles. Therefore the average speed for the trip is distance (240 miles) divided by time (5 hours) or 48 mph.

To say that the average speed for the trip is 50 mph means (by definition) that if you drove the entire way at 50 mph you would travel 240 miles in the same time as before (5 hours). But if you drive 50 mph for 5 hours you would travel 250 miles, not 240.

Resources:

1. *Simpson 's Paradox in Real Life*, Wagner,C.H.,The American Statistician (1982)

2. www.amstat.org/publications/jse/secure/v7n3/datasets.morrell.cfm

3. http://www.caltax.org/research/taxing96/taxing96.htm

YOUR 2002 HEALTH PREMIUMS September 24, 2001
http://www.federalnewsradio.com/causey/causey_092401.shtm

Health insurance premiums for 9 million federal workers, retirees and survivors will jump an average of 13.3 percent next year. In addition, Blue Cross-Blue Shield will merge its "high option" plan into its "standard" option and it will also offer a new "basic" option plan. It will have relatively low premiums, but restrict benefits to doctors and hospitals in its Preferred Provider Network.

All of this was first reported Friday by WTOP/FederalNewsRadio.Com. The Office of Personnel Management made the official announcement Friday afternoon.

Premiums went up 10.5 percent for the year 2001, 9.3 percent for 2000 and 9.5 percent for the year 1999. The highest increase, of 18.9 percent, came in the late 1980s.

Beginning in January, premiums for the "average" federal workers/retirees will go up $4.34 for self-only coverage and increase an average of $11.57 biweekly for family coverage.

Those increases show how misleading averages can be when applied to each of the 180 plus plans in the Federal Employee Health Benefits Program and its nine million customers. Some plans will increase premiums much less than the average, and a few will hold the line or even cut premiums.

Office of Personnel Management officials estimated that by moving the 80,000 plus high option Blue Cross policyholders into the standard option plan premium costs would rise about 2.5 percent. Most of the high option enrollees are older retirees.

The American Federation of Government Employees and the National Treasury Employees Union blasted OPM and Blue Cross for the higher-than-average increase in the new standard option plan.

During the open enrollment period---from November 12 to December 10---anybody in the FEHBP can switch to another plan, or option. People can also switch from fee-for-service plans to HMOs, or the other way around. There are no pre-existing clauses in the federal health program, and no waiting period for benefits.

The trend toward Health Maintenance Organizations leaving the federal program continues. More than 150 have left during the past several years, and 28 HMOs now in the program will drop out at the end of this year. But, a half dozen new HMOs will join the program.

HMO dropouts in the Washington area include the Free State Health Plan and the George Washington Health Plan with a total of about 26,000 people covered. Another HMO, CareFirst will include most of Free State's providers in an effort to get people from that plan to switch to it.

The premium increase is high---especially to federal workers anticipating a pay raise between 3.6 percent and 4.6 percent, and to retirees who will be lucky (given the low rate of inflation) to get much more than 3 percent in January. But, those increases will more than cover the increases in most health plans.

Premium Sharing: The government will continue to pay an average of 72 percent of the total premium for health plans, but the actual amount will vary between 70 percent and 75 percent. The U.S. Postal Service will continue to pay a higher share of the total

premium under the contract it has with the American Postal Workers Union, the National Association of Letter Carriers and the Mail Handlers Union.

2002 Premiums: For nonpostal employees, biweekly premiums in the Blue Cross standard self-only plan will go up $6.86 biweekly, to $41.12. Standard family coverage will rise $13.92, to $94.83 biweekly. For the same coverage, postal workers will pay $23.46 and $54.49.

As with all health plans, the government will pay the lion's share of the total premium. In the case of the Blue Cross standard family plan, for example, the total annual premium will be $318.24 next year. Of that amount, the government will pay $223.41, while the employee or retiree will pay $94.38.

Fee-for-service plans: Alliance, high option individuals will pay $60.22, an increase of $5.67 biweekly the high option family plan will go up $8.32 biweekly, to $111.72,..American Postal Workers Union, self-only up $2.06, to $48.84 and family coverage up $1.67, to $98.54...GEHA, high option self only up $9.28 to $59.70, family option up $17.14 biweekly to $119.50. GEHA's standard option premiums will remain unchanged at $27.50 and $62.50.
