Jerry Johnson
University of Nevada
July, 2002

Numeracy and Driving I
Issues of Speed

Suggested grade levels: 10 and up.

Possible subject areas: Driver's education. Health and safety. Physics or general science. Proportions.

Math skills: Definition of *average speed* as distance divided by time (r = d/t or d = r t). Proportions. It would be helpful, but not necessary, to know the definition of acceleration as speed divided by time (a = r/t or r = at)

Overview: Young people want nothing more fervently than to be able drive a car. Numeracy issues arise as soon as a novice looks at the driver's manual for the State of Nevada where he or she is confronted with the relationship between stopping distance and speed. We will discuss this as well as how much time speeding really saves.

Student activities: Numeracy and Driving, Issues of Speed

Average speed (or average rate) is defined as distance traveled divided by the time required to travel that distance.

1. Suppose you are on a 240-mile trip. If you drive the first 120 miles at 40 mph. and then the second 120 miles at 60 mph what is your average speed for the 240 - mile trip? (Assume no loss of time in changing speeds including going from 0 to 40 mph.)

2. Larry and Sue are going to drive 2 miles to a friend's house. Larry drives the first mile at 30 mph. Sue says, "We should be averaging 60 mph for this trip. You need to speed up." Larry says, "OK, if I'm going to average 60 mph I guess I should drive the second mile at 90 mph, since the average of 30 and 90 is 60." What is their actual average speed for the 2-mile trip? (Assume no loss of time in changing speeds.) How fast would Larry have to drive in order to average 60 mph for the trip?

   Now let's look at another issue: Lots of people speed these days. How much time do you save if you speed?

3. Bill drives 180 miles. The speed limit is 60 mph, but Bill drives 90 mph. (Try to estimate the answers before you calculate them, then compare the two values.)
   a) How long would it take Bill to get there at 60 mph?
   b) How long would it take Bill to get there at 90 mph?
   c) How much time would Bill save?

4. Bill drives 5 miles to work. The speed limit is 35 mph, but Bill drives 40 mph. (Try to estimate these answers before you calculate them, then compare the two values.)
   a) How many minutes would it take Bill to get to work driving at 40 mph?

b) How many minutes would it take Bill to get to work driving at 35 mph?
c) How much time would Bill save?

A page from the Nevada Driver's Manual

- The condition of the tires and brakes.
- The surface of the road.
- Your reaction time.
- How alert or how tired you are.
- The weather.

To stop your vehicle, three things have to occur:

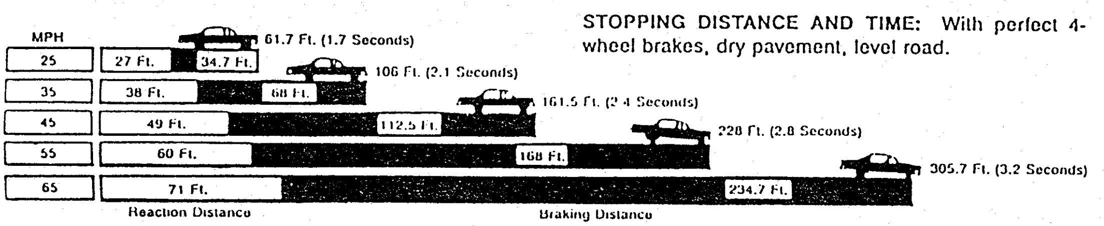
- You have to see the reason for stopping.
- Your brain has to send a message telling your foot to step on the brake pedal.
- Your foot has to move to the brake pedal and push down.

The amount of time it takes from when you see that you need to stop until you step on the brakes is called *reaction* time. Research has shown that the average driver has a reaction time of about ¾ of a second.

Stopping your vehicle also involves *braking time and distance.*

- Braking time is how much time it takes for the brakes and friction between the road and tires to stop your vehicle.
- Braking distance is how far your vehicle travels during this time.

STOPPING DISTANCE AND TIME: With perfect 4-wheel brakes, dry pavement, level road.



As you can see from the figure above, the Nevada State Driver's Manual has several references to reaction time, braking distance and stopping distance, including a bar graph showing how braking distance and stopping distance vary with speed.

Reaction time is the time required for your foot to actually move to the brake after you have decided to make a sudden stop. The driver's manual states that it is about 3/4 of a second for most people.

Reaction distance is how far your car travels while your foot moves to the brake after you have decided to stop. The faster you are going, the greater the reaction distance.

Braking distance is how far your car travels after you apply the brakes. The faster you go, the greater the braking distance.

Stopping distance is the sum of the reaction distance and the braking distance.

Jerry Johnson
University of Nevada
July, 2002

5. Both reaction distance and braking distance increase as your speed increases. Suppose you double your speed, say, from 30 to 60 mph. Based on the above bar graph, what do you think the effect will be on your reaction distance and your braking distance? Will they double as well?

While it is true that the reaction distance doubles, it may surprise you to learn that in this case your braking distance increases by a factor of four! Thus, although both reaction distance and braking distance increase as your speed increases, they do not increase in the same way.

In general, the braking distance is proportional to the *square* of the speed. That means, for example, if you double your speed your braking distance increases by a factor of four. If you triple your speed, your braking distance increases by a factor of nine!

The table below has been constructed using the data from the bar graph above.

Speed (mph)	Speed (feet per sec)	Reaction Distance (feet)	Braking Distance (feet)	Stopping Distance (feet)
20	29.3	21.7	22	43.7
25	36.7	27.1	35	62.1
30	44	32.6	50	82.6
35	51.3	38	68	106
40	58.7	43.4	89	132.4
45	66	48.8	112	160.8
50	73.3	54.3	139	193.3
55	80.7	59.7	168	227.7
60	88	65.1	200	265.1
65	95.3	70.5	234	304.5

From the bar graph one can see that reaction distance appears to be proportional to the speed. That means, for example, if the speed doubles the reaction distance doubles.

Answer the following questions using (a) the table above; (b) the fact that reaction distance is proportional to your speed; and (c) the fact that braking distance is proportional to the *square* of your speed.

6. What is the reaction distance if your speed is 80 mph? Explain how you arrived at your answer.

7. What is the braking distance if your speed is 80 mph? Explain how you arrived at your answer.

Jerry Johnson
University of Nevada
July, 2002

8. What is the stopping distance if your speed is 80 mph?

9. What is the reaction distance if your speed is 90 mph? Explain how you arrived at your answer.

10. What is the braking distance if your speed is 90 mph? Explain how you arrived at your answer.

11. What is the stopping distance if your speed is 90 mph?

For the Instructor

The solutions to problems 1 and 2 are obtained using the fact that average speed is defined as distance traveled divided by the time required to travel that distance; in symbols $r = d/t$. The usual expression of this as a formula is "distance equals rate times time" or "$d = rt$."

1. Most students will answer 50 mph, which they get by averaging the two speeds 40 and 60. This is incorrect. At a speed of 40 mph, it takes 3 hours to go 120 miles. At a speed of 60 mph, it takes 2 hours to go 120 miles. Therefore the time to make the trip is 5 hours. The average speed for the trip is distance (240 miles) divided by time (5 hours) or 48 mph.

   Here's another way to look at it: The *average* speed for the trip is such that if you drove the entire way at that speed you would travel the same distance in the same time as before. But if you drive 50 mph for 5 hours you would travel 250 miles, not 240.

2. Let's answer the second question first: How fast would Larry have to drive in order to average 60 mph for the trip? It is impossible for him average 60 mph for the trip. Notice that averaging 60 mph for the trip would require that he drive it in 2 minutes. (60 mph is a mile a minute). But he took 2 minutes driving the first mile at 30 mph, so no matter how fast he drives, he will always take more than 2 minutes to get there.

   Now the first question: What was the average speed? It takes 1/30 hours (2 minutes) to drive the first mile and 1/90 hours (40 seconds) to drive the second mile. The average speed is 2 miles divided by 1/30 + 1/90 = 4/90 hours, which is 45 mph.

3. Bill drives 180 miles. The speed limit is 60 mph, but Bill drives 90 mph.
   a) It takes Bill 3 hours to get there at 60 mph.
   b) It takes Bill 2 hours to get there at 90 mph.
   c) Thus he saves 1 hour.

Jerry Johnson
University of Nevada
July, 2002

4. Bill drives 5 miles to work. The speed limit is 35 mph, but Bill drives 40 mph.
   a) It takes Bill 5/35 hours = approximately 8.6 minutes to get there at 35 mph.
   b) It takes Bill 5/40 hours = approximately 7.5 minutes to get there at 40 mph.
   c) Thus he saves about a minute.

Discussion: In general if you drive a certain distance d at rate R in time T and the same distance at another rate r in time t then r/R = (d/t) / (d/T) = T/t. Here are three examples applying this observation:

- If you drive 60 mph how much time do you save over driving 30 mph? 30/60 = 1/2 so you get there in half the time and you save half the time.
- If you drive 60 mph how much time do you save over driving 40 mph? 40/60 = 2/3 so you get there in 2/3 of the time, which says you save a third of the time.
- In the example above, 35/40 = 7/8 so that by driving 40 mph you get there in 7/8 of the time required to get there at 35 mph, which says you save an eighth of the time.

5. Reaction distance is proportional to the speed. Thus if you double your speed you double your reaction distance. Since 80 mph is double 40 mph, the reaction distance at 80 mph is twice the reaction distance at 40 mph, or 2 x 43.4 = 86.8 or 87 feet approximately. However, *braking distance* is proportional to the *square* of the speed. Thus if you double your speed you increase your braking distance by a factor of *four*. Since 80 mph is double 40 mph, the braking distance at 80 mph is four times the braking distance at 40 mph, or 4 x 89 = 356 feet.

6. Reaction distance is proportional to the speed. Thus if you double your speed you double your reaction distance. Since 80 mph is double 40 mph, the reaction distance at 80 mph is twice the reaction distance at 40 mph, or 2 x 43.4 = 86.8 or 87 feet approximately.

7. Braking distance is proportional to the square of the speed. Thus if you double your speed you increase your braking distance by a factor of four. Since 80 mph is double 40 mph, the braking distance at 80 mph is four times the braking distance at 40 mph, or 4 x 89 = 356 feet.

8. Stopping distance is the sum of the reaction distance and the braking distance, so it is approximately 87 + 356 = 443 feet.

9. Reaction distance is proportional to the speed. Thus if you double your speed you double your reaction distance. Since 90 mph is double 45 mph, the reaction distance at 90 mph is twice the reaction distance at 45 mph, or 2 x 48.8 = 97.6 or 98 feet approximately.

You could also reason that 90 is 3 times 30, so the reaction distance at 90 mph is 3 times the reaction distance at 30 mph, or 3 x 32.6 = 97.8 feet. Note that this answer is not the same as the previous one (97.6) but they round off to the same nearest foot (98). Also, there is always error in physical measurements, and an error of 0.2 in a measurement of 98 is very small - about 2 tenths of one percent.

10. Braking distance is proportional to the square of the speed. Thus if you double your speed you increase your braking distance by a factor of four. Since 90 mph is double 45 mph, the braking distance at 90 mph is four times the braking distance at 45 mph, or 4 x 112 = 448 feet.

You could also reason that 90 is 3 times 30, so the braking distance at 90 mph is 9 times the braking distance at 30 mph, or 9 x 50 = 450 feet approximately. Note that this answer is not the same as the previous one (448) but an error of 2 in a measurement of 448 is very small - about ½ of one percent.

11. Stopping distance is the sum of the reaction distance and the braking distance, so it is approximately 98 + 448 = 546 feet (or 98+450= 548 using the second way).

Discussion: Why is braking distance proportional to the square of the speed? Of course you can slam on the brakes at different speeds and measure the distances to verify it, but here's a sketch of the physics behind it: First $d = r\,t - \frac{1}{2}\,a\,t^2$ is how far you travel in time t if your initial rate is r and your deceleration is constant at $-a$. If your initial rate is r and your deceleration is constant at $-a$, then $r - a\,t$ is how fast you are going at time t. When you have stopped, $r - at = 0$, so $t = r/a$. Substitute this for t in $d = r\,t - \frac{1}{2}\,a\,t^2$ to get $d = r\,(r/a) - \frac{1}{2}\,a\,(r/a)^2 = \frac{1}{2}\,r^2/a$.

Resources:

CHANCE News 8.09 (October 7, 1999 to November 16, 1999) contains a lot of interesting material about transportation safety issues, including a discussion of why air travel is safer than automobile travel. The link is:
www.dartmouth.edu/~chance/chance_news/recent_news/chance_news_8.09.html#fear overtakes logic