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Lotteries

Suggested grade levels: 12 to 14

Math skills: combinations, probability

Possible subject area: economics, probability & statistics

Overview

State lotteries are currently operating in approximately 3/4 of the states in the U.S. These lotteries boast of multi-million dollar jackpots to entice people to buy tickets and take a shot at becoming rich. Clearly, a prize of this magnitude is tempting, but what are the chances of actually winning this money? How much money will you receive if you win? Is a lottery jackpot "due" to be hit if it has not been won several times in a row? For those who don't want to spring for the price of a lottery ticket, Publisher's Clearing House offers a free shot at $1,000,000 with its PCH Lotto, and of course, no purchase is required!

Lotteries

SUPERLOTTO PLUS JACKPOT RAISES TO $120 MILLION - THIRD LARGEST JACKPOT IN CALIFORNIA LOTTERY HISTORY

Sacramento, August 23, 2002 -- Lottery officials announced the estimated SuperLotto Plus jackpot for Saturday night's draw at $120 million. "Sales are great and those sales have allowed us to push the jackpot up to $120 million, which makes it the third highest jackpot in our history," said Joan Wilson, Director of the California Lottery. (http://www.calottery.com/newsflash/newsflash.asp)

A jackpot winner can choose to receive the entire amount in 26 yearly payments or elect to take a single lump sum payment. A $120 million winner would receive 26 annual payments with the first payment equaling approximately $3 million and the 26th payment being more than $6 million before taxes. If the winner decided to take a lump sum payment, the cash value would equal roughly half of the jackpot, $60 million. The Lottery would invest the balance - another $60 million - to pay out the jackpot prize over 26 years. (http://www.calottery.com/newsflash/newsflash.asp)

Four winning tickets were sold for this $120 million dollar drawing. Using the information provided:

1. Approximately how much money would each winner receive before taxes if they decide to take a lump sum payment?

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2. Approximately how much money would each winner receive in the first year, if they choose to take their money over a 26 year period?

3. Approximately how much money would each winner receive in the twenty-sixth year, if they choose to take their money over a 26 year period?

Another important consideration is taxes. Any winner of a large amount of money is going to pay a hefty chunk, probably about 1/3 or more to federal and state taxes.

4. Approximately how much money will each winner have after taxes if an immediate lump sum payment is chosen?

When it comes to thinking about our chances of winning a jackpot like the one we have discussed, rationally, we are aware that the chances are extremely slim. Nevertheless, when the jackpots get large, long lines of people purchasing tickets form.

First let's consider the actual mathematical probability of winning. Most state lotteries include a variety of games. In most cases the premier games involve correctly selecting a certain number of numbers from some larger number of possible numbers. For illustrative purposes, we can consider Florida Lotto, the premier game offered by the Florida State Lottery.

In this game, the bettor must correctly select 6 numbers from 53 possibilities. The order in which the numbers are selected does not matter as long as in the end the correct 6 numbers have been drawn. The mathematical term for this situation is a combination. Before considering this probability, let's consider a much simpler example. Suppose a prize is being offered if you correctly select 2 of 5 possible numbers. When the first number is selected, 2 of the 5 numbers are favorable. If the first number is favorable, 1 of the 4 remaining numbers is favorable for the second drawing. The probability of two consecutive favorable drawings can be found by multiplying 2/5 times 1/4, yielding 1/10 as the probability of winning this game.

There is another way of finding the probability of winning our hypothetical game.

5. See if you can list all of the possible combinations of 2 numbers that can be chosen from the five numbers, 1, 2, 3, 4, 5.

Think carefully about the following question.

6. What is the probability of correctly selecting 3 numbers out of 5? Do you agree that this must be harder than correctly selecting 2 numbers out of 5? It certainly seems it should be. OK, go ahead and calculate this probability by either of the methods previously used OR come up with another way of finding it.

Returning to the probability of winning the Florida Lotto, we must multiply (6/53)(5/52)(4/51)(3/50)(2/49)(1/48). This product is 1/22,957,480, making your chances of correctly selecting the 6 numbers drawn at about 1 chance in 23 million! It should be

noted that prizes are awarded for matching 5 of 6, 4 of 6, or 3 of 6 numbers in this game, but our concern is only with winning the big prize.

7. Is there another number of correct selections the Florida Lotto could ask bettors to make that would also yield a probability of 1/22,957,480 of winning?

Luckily, the onus of determining the probability of winning does not lie with the bettor. Most if not all lotteries provide this information. With the evolution of the web, most of this information can be found on-line. The probabilities we have considered for the Florida Lotto can be found at http://www.floridalottery.com.

There are many fallacies that exist regarding the playing of lotteries. One common fallacy is that if the jackpot is not hit several times in a row, it is due to hit. This is not the case. Each time the numbers are selected, the probability of any combination occurring remains the same, 1/22,957,480. However, when the jackpot gets large, a much larger number of tickets are sold as people are drawn in by a bigger potential payoff.

8. Would you be correct in reasoning that whenever the Florida Lotto jackpot is more than $23 million you should play since you are getting good odds for your money?

The overall likelihood of the jackpot being hit increases when the jackpot is large due to the increased number of tickets usually sold when this is the case. The probability of any specific combination winning however remains the same.

We might also consider the Lottery money from a different perspective. Why do you think states run lotteries? You may be surprised to learn that their primary reason is NOT to provide entertainment to their citizens. Instead, the main purpose of a lottery is to raise money for the state. So, one could correctly argue that if the state is making money, the bettors are not being provided with a game in which they can expect to win in the long run.

Approximately 30 to 38 percent of the money spent on lottery tickets goes to funding state projects, approximately 50% is returned in the form of winnings and the remainder is used to fund the operation of the lottery. In New York for example, 1.35 billion dollars was provided to state education funds from lottery revenues in fiscal year 1999-2000 (http://www.nylottery.org/).

You have probably heard of the Publisher's Clearinghouse sweepstakes, often paid during the Super Bowl. Publisher's Clearinghouse, also, has a daily game called PCH Lotto in which a contestant can win a $1,000,000 jackpot. In this game, 7 of 50 numbers must be selected correctly in order to win the grand prize.
(http://play.pchlotto.com/play?action=rules.html&site=PCH)

9. Do you think it is more likely to win the grand prize in PCH Lotto than it is to win a jackpot in the Florida Lotto?

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For the Teacher

Answers

1. Since there are 4 winners, each would get approximately $30,000,000 if they took the money over a 26 year period. The lump sum payment is about half of the jackpot amount, making the approximate one time payment $15,000,000.

2. The first payment would be 1/40 of the $30,000,000 share. Thus each would get approximately $750,000 in the first of 26 payments.

3. The twenty-sixth payment would be 1/20 of the $30,000,000 share. Thus each would get approximately $1,500,000 in the final payment.

4. Each winner would retain approximately 2/3 of the $15,000,000 lump sum payment or approximately $10,000,000.

5. If you listed the following combinations you are correct: 1-2, 1-3, 1-4, 1-5, 2-3, 2-4, 2-5, 3-4, 3-5, 4-5. Since exactly 1 of these 10 combinations of numbers will be selected, the probability of any one of them occurring is 1/10, as before.

6. By the probability method we would multiply 3/5 * 2/4 * 1/3 (be sure you understand why). Surprisingly, this product yields 1/10, the same probability of correctly selecting 2 numbers out of 5.

Now let's list all possible combinations of 3 numbers that could occur. We find 3-4-5, 2-4-5, 2-3-5, 2-3-4, 1-4-5, 1-3-5, 1-3-4, 1-2-5, 1-2-4, 1-2-3. Ten combinations of 3 numbers selected from 5, verifying the probability of 1/10. Do you see why these combinations have been listed in this order?

You may have noticed that the 3 number combinations listed in the previous paragraph have been formed by considering the numbers NOT included in each of the 2 number combinations listed earlier. Thus, there is a one-to-one correspondence between 2 number combinations of 5 and 3 number combinations of 5. For this reason, the probability of correctly selecting either 2 numbers out 5 and the probability of correctly selecting 3 numbers out of 5 are equal.

7. Yes, of course! You could be asked to correctly select 47 of the 53 numbers correctly. You can understand why by comparing these situations to the probabilities of correctly selecting 2 or 3 numbers correctly from 5 numbers. Naturally, it would be absurd to ask people to select 47 of 53 as many potential players may feel that filling in the form is not worth winning millions of dollars. Well, not exactly, but obviously selecting 6 is more convenient!

8. The answer is NO! The increased jackpot amount will probably mean that more tickets will be sold. It is also possible that more than one person may correctly select all 6 numbers drawn. If this is the case, the winners will equally split the total jackpot.

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Consider the following hypothetical situation in the Florida Lotto. When the jackpot is relatively low, little interest is generated and maybe only 2 million tickets are sold. The overall probability that there will be a winner is less than 1 in 11. Now several weeks later a huge jackpot has accrued and the lines to buy tickets are long. Let's say that $50 million tickets are sold. Clearly, it is very likely that at least one other person has selected the same 6 numbers that you have. Thus, if you win, the jackpot will not be entirely yours but will be split among all who chose the correct combination.

9. There are only 50 numbers to choose from not 53 which would make winning in the PCH Lotto somewhat easier. Having to correctly pick 7 numbers rather than 6 more than makes up for this difference, however. It is left to the interested reader to prove the probability as stated on the Publisher's Clearing House website of 1 in 99,884,400.

References

1. http://www.calottery.com/
2. http://www.floridalottery.com
3. http://www.nylottery.org/
4. http://www.pch.com/
5. http://www.state.nh.us/lottery/nhlotto.htm
6. Quinn, R. J. (2002). State Lotteries. In Mathematics, ed. Barry Max Brandenberger, Jr. New York: Macmillan Reference USA.