max # matching

Theorem. *Let G be a bipartite graph. Then $\nu(G) = \tau(G)$.* — min # v. cover

Proof. Let G be a minimal counterexample. Then G is connected, is not a circuit, — bipartite. nor a path. So, G has a node of degree at least 3. Let u be such a node and v one of its neighbors. If $\nu(G\backslash v) < \nu(G)$, then, by minimality, $G\backslash v$ has a cover W' with $|W'| < \nu(G)$. Hence, $W' \cup \{v\}$ is a cover of G with cardinality $\nu(G)$ at most. Assume, therefore, there exists a maximum matching M of G having no edge incident at v. Let f be an edge of $G\backslash M$ incident at u but not at v. Let W' be a cover of $G\backslash f$ with $|W'| = \nu(G)$. Since no edge of M is incident at v, it follows that W' does not contain v. So W' contains u and is a cover of G. ∎

The same proof easily extends to Egerváry's generalization [2] of König's result to graphs with nonnegative weights on the edges.

They construct the cover from the matching by
- taking one vertex from each edge in the matching
- Since v is not incident to any edge in M, v is not in the cover. To cover the edge uv, u must be in the cover. So f is covered.