Exponentially-convergent simulations of extreme-mass-ratio binary black hole systems: A discontinuous Galerkin method for the Teukolsky equation with singular source terms

Scott Field

Gravitational wave signals from extreme mass ratio binary (EMRB) systems are a crucial target for space-based gravitational wave detectors. In such systems, a smaller black hole orbits around a supermassive black hole. These systems are typically modeled as a distributionally-forced Teukolsky equation that describes the evolution of spacetime metric perturbations and gravitational radiation emitted to the far-field. This equation poses two computational challenges: (i) the source terms feature not only a Dirac delta distribution but also its first, second, and third derivatives and (ii) we need to supply appropriate outer boundary conditions (conditions of complete transparency) and access the far-field signal on a finite computational domain. In this talk, we describe a multi-domain discontinuous Galerkin method for this problem. To handle the Dirac delta, we expand the solution in spherical harmonics and recast the sourced Teukolsky equation as a first-order system by using distributional auxiliary variables. This allows us to derive the method's numerical flux to correctly account for the distributional right-hand side and achieve spectral convergence even at the location of the distributional source terms. To connect the near-field to the distant far-field, we use the hyperboloidal layer method, allowing us to supply outer boundary conditions and provide direct access to the gravitational wave signal. We also observe superconvergence in the extracted gravitational waveform. Several benchmark numerical examples will be presented. Linear wave equations with source terms containing Dirac delta functions and their derivatives occur in other areas of engineering and physics, and generalizations of our method to other PDEs sourced by arbitrary nth-order derivatives of a Dirac delta distribution will also be discussed.