Trefftz and quasi-Trefftz methods for time-harmonic wave propagation and beyond

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In the field on numerical partial differential equations (PDEs), many methods rely on standard bases, like polynomial bases, but other methods rely on problem-dependent bases. Trefftz methods rely, in broad terms, on the idea of approximating solutions to PDEs via Galerkin-type formulations discretized with basis functions solving exactly the governing PDE locally, making explicit use of information about the ambient medium. They are of particular interest for wave propagation problems, for example with plane wave bases with the wave number depending on the propagation medium. The presentation will start with an introduction to Trefftz methods. However, in general, for problems modeled by PDEs with variable coefficients, no exact solutions are available. Hence quasi-Trefftz methods have been introduced to address this problem: they rely not on exact solutions to the PDE but instead on high order approximate solutions constructed locally. We will discuss some of the fundamental properties of these numerical methods for problems governed by the Helmholtz equation. Finally we highlight the main difficulty in developing quasi-Trefftz methods for time-harmonic electromagnetic wave propagation, and present recent developments to problems governed by a class of elliptic equations.