Wavelength Assignment and Generalized Interval Graph Coloring

(Short Form Abstract)

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Abstract

In this paper we study wavelength assignment on an optical linesystem without wavelength conversion. Consider a set of undirected demands along the line. Each demand is carried on a wavelength and any two overlapping demands require distinct wavelengths. Suppose μ wavelengths are available in the system. We define $\ell(e)$, the load on link e, to be the smallest integer such that $\ell(e)\mu$ is at least the number of demands passing through e. Hence, $\ell(e)$ is the minimum number of fibers required on e in order to support all demands.

We present a polynomial-time wavelength assignment algorithm that guarantees each wavelength appears at most $\ell(e)$ times on each link e. (This generalizes the well-known fact that interval graphs are perfect.) In the presence of MOADMs (mesh optical add/drop multiplexers), devices that multiplex distinct wavelengths from different fibers into a new fiber, we only need to deploy $\ell(e)$ fibers per link. On the other hand, if each demand has to stay on a single fiber, as is the case without MOADMs, we show that some links may require more than $\ell(e)$ fibers. In fact, we show that it is NP-complete to decide if a set of demands can be carried on a given set of fibers, or if there exists a set of fibers with a given total length that can carry all the demands.

1 Introduction

Wavelength division multiplexing (WDM) partitions the bandwidth on an optical fiber into multiple channels, each of which operates at a distinct wavelength [6]. For example, a standard single mode fiber can carry 128 wavelengths at a data rate of 10 Gbps per wavelength. In a *transparent* network signals pass optically from link to link without changing wavelengths, thus avoiding the expense and delay of "O-E-O" (optical to electrical to optical) conversion. However, one must be careful to assign wavelengths to signals so that no two signals on the same fiber on a given link are on the same wavelength. This problem, often combined with routing of demands, is much studied (see e.g. [1, 2, 7, 8, 9, 10]) and very difficult in general.

However, one methodology in optical network design is to decompose the network into a collection of lines, called *optical linesystems* [3]. Each linesystem is transparent, but between linesystems signals undergo O-E-O conversion; thus the problem of assigning wavelengths to signals can be tackled independently in each linesystem, and of course there is no routing issue in a line.

For a linesystem with μ wavelengths, and only one fiber per link, the wavelength assignment problem is equivalent to interval graph coloring (see e.g. [4]). As long as a maximum of μ demands (undirected signal requests) pass through each link, an assignment is always possible.

A device called a *mesh optical add/drop multiplexer* (MOADM) allows linesystems containing multi-fiber links to move signals on different fibers to the same fiber, or vice-versa, as they pass through a node—provided they are on different wavelengths. When MOADMs are present, a wavelength assignment need only ensure that the number of signals of a given wavelength passing through a given link is no greater than the number of fibers in that link; the issue of which signal travels on which fiber is unimportant.

We will show that in this case, a wavelength assignment is again always possible as long as the number of demands through a link does not exceed μ times the number of fibers in the link. Moreover, we provide a polynomial-time algorithm for finding such an assignment. It follows that when designing such a system, we can get by with just the amount of fiber required by the above condition.

Without MOADMs, the linesystem may be regarded as a collection of fibers each of which occupies an interval of the line. Each signal must then be assigned not only a wavelength but also a fiber which covers the intended interval. We show that this problem is NP-complete. If $\mu > 1$ more fiber is in general required than in the MOADM case, and in fact deploying fiber so as to satisfy the demands but minimize total length is also NP-hard.

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2 Problems and Results.

Consider a linesystem of n links $e_1, e_2, \ldots, e_n$, with link e_i carrying f_i fibers. We represent a demand by $[i, j]$ for $i \leq j$ if it requires links $e_i, \ldots, e_j$; the set of demands will be denoted by D. The load $\ell_i := \ell(e_i) := \lceil d_i/\mu \rceil$ on link e_i is the minimum number of fibers required to carry all the demands on link e_i, where d_i is the number of demands on link e_i and μ is the number of wavelengths.

Our problem is to assigns wavelengths to demands in such a way that the number of demands through e_i of a given wavelength never exceeds f_i. We call this GENERALIZED INTERVAL GRAPH COLORING.

THEOREM 2.1. *GENERALIZED INTERVAL GRAPH COLORING is solvable in time polynomial in n, μ, $\sum f_i$ and $|D|$; and a solution always exists when $\ell_i \leq f_i$ for each i.*

Proof. Roughly speaking, the proof proceeds by induction on the total number of link-fibers. The number of fibers on a link e_i for which $f_{i-1} \leq f_i \geq f_{i+1}$, not all three equal, can be reduced; and the problem is solved easily when no such link exists.

As an example of an instance of GENERALIZED INTERVAL GRAPH COLORING, take $n = 3$, $\mu = 2$, $D = \{[1,3],[1,2],[2,3]\}$, $f_1 = f_3 = 1$, $f_2 = 2$. We can assign $[1,3]$ "green" and the other demands "red".

Without MOADMs, the linesystem consists of a collection F of fiber intervals; a set D of demands can be satisfied iff they can be packed into the fiber intervals so that no link in a fiber interval is carrying more than μ demands. We call this problem PACKING INTERVALS IN INTERVALS. Note that the demand set in the above example cannot be handled either with $F = \{[1,3],[2,2]\}$ or $F = \{[1,2],[2,3]\}$; more fiber is required.

THEOREM 2.2. *For any fixed $\mu \geq 1$, PACKING INTERVALS IN INTERVALS is NP-complete.*

Proof. By reduction from CIRCULAR ARC GRAPH COLORING (see [5]). In the $\mu = 1$ case, the circle is cut at an arbitrary node to make an interval graph. D consists of the unsplit arcs, and F contains the complement of each split arc plus enough full-length intervals so that $|F|$ is the required number of colors.

Finally, if only the demands D are given, together with link lengths, we would like to be able to construct a set F of fiber intervals of minimum total length which can satisfy D; we call this problem FIBER MINIMIZATION. In the presence of MOADMs this problem is easy since we have seen that ℓ_i fibers at link e_i are necessary and sufficient. However, here,

THEOREM 2.3. *FIBER MINIMIZATION is NP-complete for any fixed $\mu \geq 2$.*

Proof. By reduction from PACKING INTERVALS IN INTERVALS. In the $\mu = 2$ case the line is greatly extended; the demands for FIBER MINIMIZATION consist of those from PACKING INTERVALS IN INTERVALS plus two much longer demands for each fiber interval, the longer of which is the disjoint union of the shorter and the given fiber interval.

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Appendix

Proof of Theorem 2.1: We prove a stronger statement. We show how to assign wavelengths so that each wavelength appears at most ℓ_i times but at least $\lfloor d_i/\mu \rfloor$ times on link e_i. (Recall d_i is the number of demands through e_i.) If the link loads are upside-down unimodal, i.e. $\ell_1 \geq \ldots \geq \ell_k \leq \ldots \leq \ell_n$ for some $1 \leq k \leq n$, then GENERALIZED INTERVAL GRAPH COLORING (GIGC) is easy. We assign wavelengths like coloring an interval graph starting from link e_k and expanding to both. This is the basis of our induction.

We call link e_i a *peak* if $\ell_i \geq \max\{\ell_{i-1}, \ell_{i+1}\}$ and $\ell_i > \min\{\ell_{i-1}, \ell_{i+1}\}$. If there is no peak, the link loads are upside-down unimodal. For a peak at e_i, let P be the set of demands that pass through link e_{i-1} *or* e_i *or* e_{i+1}, and let Q be the set of the remaining demands. We first assign a number in the range $1, \ldots, \mu\ell_i$ to each demand in P. We then create a new set of demands P' so that demands in P that are assigned the same number become one demand in P'. We make sure that under $P' \cup Q$ the link load is unchanged on all links except that the load on e_i is decreased. Inductively, we solve GIGC on demand set $P' \cup Q$. Finally, we map the wavelength assignment of P' to P, thereby solving the original problem on $P \cup Q$.

We assign a number in the range $1, \ldots, \ell_i\mu$ to each demand in P such that each number appears at most once on each link. Let A be the set of numbers that appear both on links e_{i-1} and e_{i+1} and let B be the set of the remaining numbers on e_{i-1} and C be the remaining on e_{i+1}. We now create a set of demands P' of which $|A|$ pass through link e_i. For each number $x \in A$ we identify all the demands that are assigned x and hook them up for P'. For example, if $[j, i-1]$ and $[i+1, k]$ are assigned x then we create a demand $[j, k]$ for P'; if $[j, k]$ is assigned x where $[i-1, i+1] \subseteq [j, k]$ we also put $[j, k]$ in P'. If a demand is assigned a number $x \in B \cup C$, then we shorten it, if necessary, so that it does not pass through e_i. For example, if $[j, i]$ is assigned $x \in B$ where $j \leq i-1$ then we create a demand $[j, i-1]$ for P'.

Let us bound $|A|$. Without loss of generality, we assume $|A| + |B| = \ell_{i-1}\mu$ and $|A| + |C| = \ell_{i+1}\mu$. (Otherwise we can add extra demands $[i-1, i-1]$ and $[i+1, i+1]$ to make up the difference.) We also have $|A| + |B| + |C| \leq \ell_i\mu$. This implies $\ell_{i-1} + \ell_{i+1} - \ell_i \leq |A|/\mu \leq \min\{\ell_{i-1}, \ell_{i+1}\}$. Since the upper and lower bounds on $|A|/\mu$ are integers, we derive

$$\lceil |A|/\mu \rceil \leq \min\{\ell_{i-1}, \ell_{i+1}\} < \ell_i \quad (2.1)$$

$$\lfloor |A|/\mu \rfloor \geq \ell_{i-1} + \ell_{i+1} - \ell_i. \quad (2.2)$$

By (2.1) the link load on e_i is reduced under the demand set $P' \cup Q$. The load on all other links are unchanged. We inductively solve GIGC on $P' \cup Q$.

For each demand in P' we assign its corresponding demand(s) in P with the same wavelength. It is easy to see that each wavelength appears ℓ_j times on link e_j for $j \neq i$. It remains to argue for e_i. GIGC on $P' \cup Q$ guarantees that the numbers in $A \cup B$ are mapped to the same wavelength ℓ_{i-1} times, the numbers in $A \cup C$ are mapped to the same wavelength ℓ_{i+1} times and the numbers in A are mapped to the same wavelength at least $\lfloor |A|/\mu \rfloor$ times. By (2.2), we know the numbers in $A \cup B \cup C$ are mapped to the same wavelength at most ℓ_i times. The only demands that are not yet assigned wavelengths are those in P of the form $[i, i]$. Each of them can take a wavelength that appears fewer than ℓ_i times on e_i.

To bound the running time, we see that solving demand set $P \cup Q$ requires solving $P' \cup Q$, assigning numbers to P, constructing P' from P and mapping numbers to wavelengths. The last three are clearly polynomial-time operations. Since from $P \cup Q$ to $P' \cup Q$ the total load is reduced, the number of such reductions is polynomial. Hence, GIGC has a polynomial-time solution.