# Cavalieri's Principle

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Cavalieri's principle states the following: If two plane regions are bounded between two lines, and if for every horizontal line the intersection of this line with the first plane region has the same length as the intersection of this line with the second plane region, then both regions have the same area.

1. Suppose we have a stack of coins where every coin is directly on top of the previous one. If we perturb the coins a bit so they do not sit evenly on top of each other, does the total volume of all of the coins change?

2. Let's prove Cavalieri's principle with calculus. If $f, g : [a, b] \to \mathbb{R}$ are continuous functions, how can we express the *area between $f$ and $g$*?

3. If $h : [a, b] \to \mathbb{R}$ is continuous, and if $F, G : [a, b] \to \mathbb{R}$ are defined by $F(x) = f(x) + h(x)$ and $G(x) = g(x) + h(x)$, what is the area between $F$ and $G$?

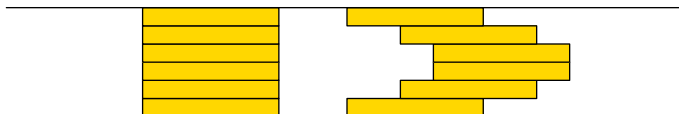4. Prove your formula using the rules of integration.
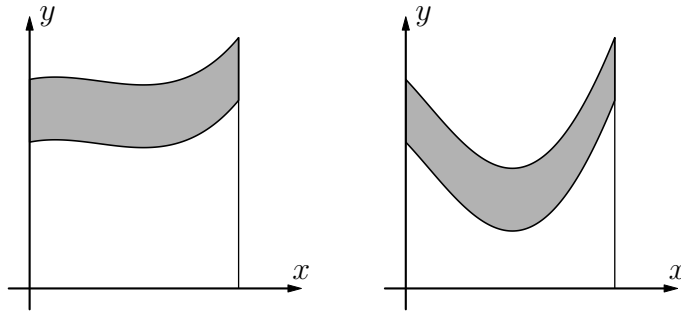


Figure 1: Visual for Cavalieri's Principle

Figure 2: Visual for the Area Between Curves