Escape Velocity

Ryan Maguire

September 29, 2023

If you throw a ball in the air, it will come back down. *Or will it?* There is a certain height the ball will get where at that point it will momentarily stop moving, before falling back down towards the earth. What if you threw the ball so hard that this height was *infinity*? The idea is called escape velocity. The laws of Newtonian mechanics tell us that the potential and kinetic energy of a ball being thrown are:

$$U + V = -\frac{GMm}{r} + \frac{1}{2}mv^2 \tag{1}$$

Here, G is a universal constant, M is the mass of the Earth, m is the mass of the ball, r is the distance from the core of the Earth to the ball, and v is the speed of the ball. The law of conservation of energy says that the sum of kinetic and potential energy is conserved:

$$U_0 + V_0 = U_1 + V_1 \tag{2}$$

Where U_0 and V_0 are the initial potential and kinetic energy, respectively, and U_1 and V_1 are the kinetic and potential energies at infinity. The initial values are given by:

$$U_0 + V_0 = -\frac{GMm}{R} + \frac{1}{2}mv_0^2 \tag{3}$$

Here, R is the radius of the Earth, and v_0 is the initial speed of the ball.

1. Try to find the formula for escape velocity. You want to solve the values of U_1 and V_1 *at infinity.* Remember, we want the ball to reach zero velocity at infinity.

2. What would happen if you threw the ball with a speed *greater* than escape velocity? What would the final speed be *at infinity* (Again, you want to solve the limit at infinity for V_1)?

3. Using the following quantities, what is escape velocity for Earth?

    - $M = 5.9722 \times 10^{24}\text{kg}$
    - $G = 6.674 \times 10^{-11}\text{m}^3\text{kg}^{-1}\text{s}^{-2}$

- $R = 6.3781 \times 10^6$m

4. What would happen if M was so big that escape velocity was greater than the speed of light?

5. The speed of light is 2.99792458×10^8m/s. How big would the radius of Earth need to be for it to turn into a black hole?