Escape Velocity - Solution

Ryan Maguire

September 29, 2023

1. We want $v_1 = 0$, the speed at infinity. Since the potential is $U = -GMm/r$, the limit at infinity for this is zero. So, $U_1 + V_1 = 0$ at infinity. By conservation of energy, $U_0 + V_0 = 0$, so $GMm/r = \frac{1}{2}mv_0^2$. Canceling little m, we get $v_0^2 = 2GM/r$. So:

$$v_0 = \sqrt{\frac{2GM}{R}} \tag{1}$$

2. The limit of U is still zero at infinity, so we'd have:

$$v_1^2 = v_0^2 - \frac{2GM}{R} \tag{2}$$

   Taking square roots of both sides gives us the speed at infinity:

$$v_1 = \sqrt{v_0^2 - \frac{2GM}{R}} \tag{3}$$

3. Just plug in the values for $\sqrt{2GM/R}$. You get 11,186 meters per second, or about 25,000 miles per hour.

4. This is the definition of a black hole.

5. Now solve for r instead. We have:

$$c = \sqrt{\frac{2GM}{r}} \tag{4}$$

   Where c is the speed of light, G and M are the same. The radius Earth would need to be is:

$$r = \frac{2GM}{c^2} \tag{5}$$

   Plugging in the numbers, we get about a centimeter. So, smaller than a pea.