What Color is the Sun?

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I think most of us are convinced that (in the year 2021 C.E.) the sun is yellow. If you're reading this in 4 billion years, congratulations! The sun has probably swallowed the Earth and is now redder.

Let's use calculus to derive the fact that the sun is yellow. Planck's law says that the spectrum of a black-body (which the sun is) is given by the following horrendous formula:

$$u(\lambda) = \frac{2hc^2}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda kT}\right) - 1} \tag{1}$$

Fortunately, most things here are constant. h, c, and k are universal constants known as Planck's constant, the speed of light, and Boltzmann's constant, respectively. T is the temperature of the sun, and λ is the wavelength of light given off. $u(\lambda)$ measures how much of a certain wavelength is emmitted. So if we want to know what the temperature of the sun is, we are simply asking where does $u(\lambda)$ hit its *absolute maximum*. We need to differentiate u and solve for zero. Becuase u is so messy, the work is very tedious, but it only uses the rules of differentiation. The chain rule, the quotient rule, and the power rule.

At the end of our messy calculation, we'd get the following:

$$\lambda_{max} = \frac{b}{T} \tag{2}$$

where b is a constant called *Wien's displacement constant*. This formula is known as *Wien's law*. The value b is roughly:

$$b = 2.897771955 \times 10^{-3}\text{mK} \tag{3}$$

mK means *meters-Kelvin*. The sun has a surface temperature of 5,772K, so plugging this into our formula we get:

$$\lambda_{max} = \frac{2.897771955 \times 10^{-3}}{5,772}\text{meters} = 5.0203949 \times 10^{-7}\text{meters} \tag{4}$$

Perfect yellow is 5.6×10^{-7} meters, so our sun slightly greener than true yellow. But still, pretty yellow.

As a final question, what star is hotter, a blue star, or a red star? Our intuition on Earth says that red is hotter. Red is the color of fire, blue is the color of water, so red should be hotter. Wein's law says otherwise. If we have a very large temperature, we get a very small wavelength, and blue has a smaller wavelength than red. So the hottest stars in the sky are the blue and blue-white ones.