Domains of Functions - Example 5

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September 28, 2023

Consider the following function:

$$f(x) = \frac{\sin(x)}{x} \tag{1}$$

This function is well-defined everywhere except for $x = 0$. However, the *small angle approximation*, often used by physicists, states that if x is a small real number, then $\sin(x) \approx x$. The symbol $\approx$ means *is approximately equal to*. We can verify this from the graph of the two functions close to the origin. By examining Fig. 1 we see that for small values the graphs of $\sin(x)$ and x are nearly identical. Using this we have, for small x, the following:

$$\frac{\sin(x)}{x} \approx \frac{x}{x} = 1 \tag{2}$$

And indeed the *limit* of $f(x)$ as x approaches zero is 1, even though $f(0)$ is undefined. With this we can define a new function by *filling in* where $f(x)$ is undefined. This is the *sinc* function, and it's use is widespread in physics, engineering, and signal processing.

$$\operatorname{sinc}(x) = \begin{cases} \frac{\sin(x)}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases} \tag{3}$$

Since the limit of $\operatorname{sinc}(x)$ as x approaches zero is 1, and since $\operatorname{sinc}(0) = 1$, we have from the *limit definition of continuity* that $\operatorname{sinc}(x)$ is continuous at 0. This function is shown in Fig. 2.

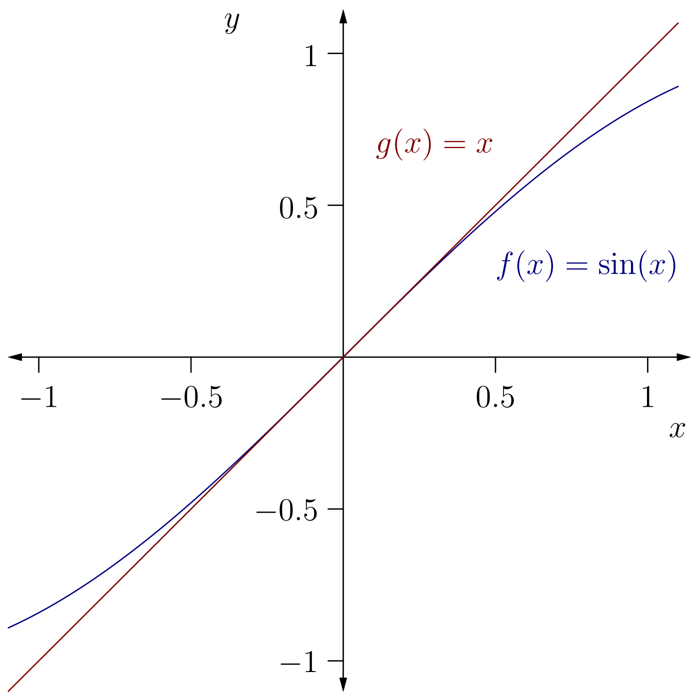

Figure 1: Small Angle Approximation

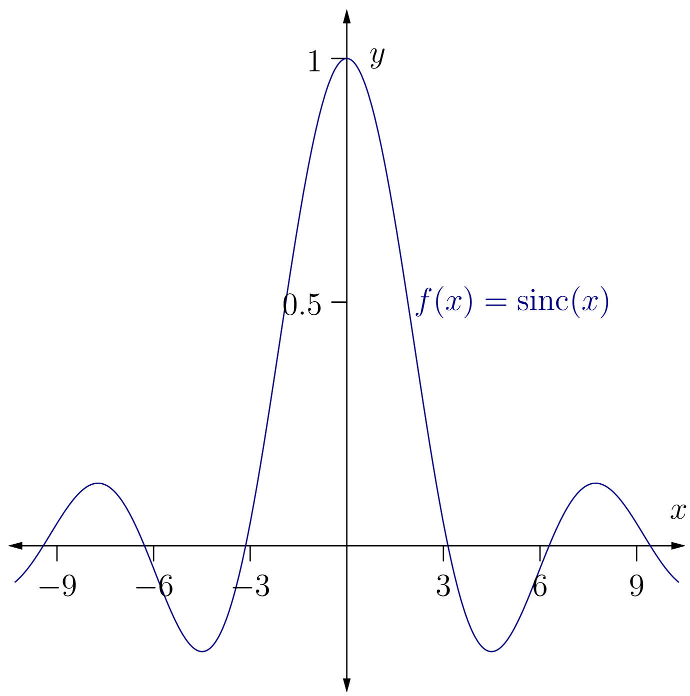

Figure 2: The sinc function