

# Domains of Functions - Example 5

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Consider the following function:

$$f(x) = \frac{\sin(x)}{x} \tag{1}$$

This function is well-defined everywhere except for  $x = 0$ . However, the *small angle approximation*, often used by physicists, states that if  $x$  is a small real number, then  $\sin(x) \approx x$ . The symbol  $\approx$  means *is approximately equal to*. We can verify this from the graph of the two functions close to the origin. By examining Fig. 1 we see that for small values the graphs of  $\sin(x)$  and  $x$  are nearly identical. Using this we have, for small  $x$ , the following:

$$\frac{\sin(x)}{x} \approx \frac{x}{x} = 1 \tag{2}$$

And indeed the *limit* of  $f(x)$  as  $x$  approaches zero is 1, even though  $f(0)$  is undefined. With this we can define a new function by *filling in* where  $f(x)$  is undefined. This is the *sinc* function, and it's use is widespread in physics, engineering, and signal processing.

$$\text{sinc}(x) = \begin{cases} \frac{\sin(x)}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases} \tag{3}$$

Since the limit of  $\text{sinc}(x)$  as  $x$  approaches zero is 1, and since  $\text{sinc}(0) = 1$ , we have from the *limit definition of continuity* that  $\text{sinc}(x)$  is continuous at 0. This function is shown in Fig. 2.

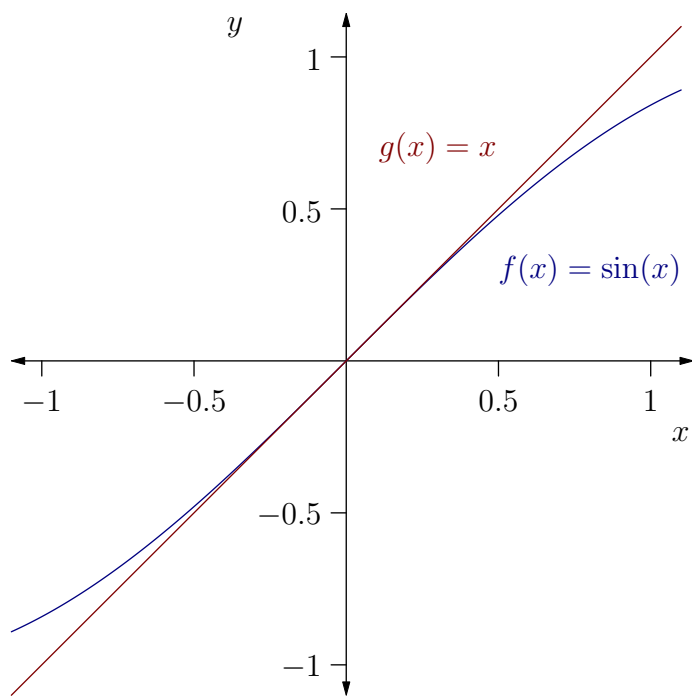


Figure 1: Small Angle Approximation

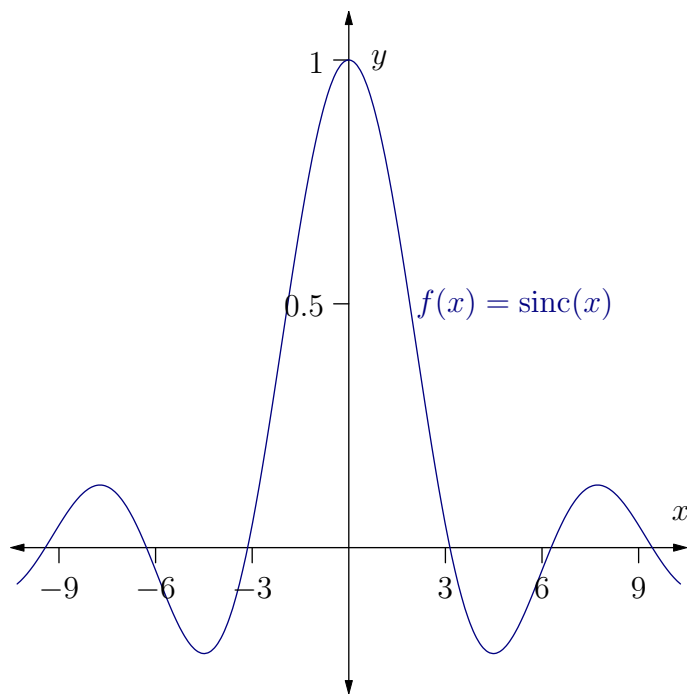


Figure 2: The sinc function

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