

# Domains of Functions - Example 6

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Consider the expression:

$$f(x) = \frac{3x}{\frac{2}{x} - 1} \quad (1)$$

For what real numbers is this well-defined? The only thing to look out for is division-by-zero. The denominator is  $\frac{2}{x} - 1$ , so if we are to avoid division-by-zero we must exclude  $x = 2$ . The expression  $\frac{2}{x}$  also contains a division, and so we must also exclude  $x = 0$ . In set theory notation, we can write the domain as:

$$D = (-\infty, 0) \cup (0, 2) \cup (2, \infty) \quad (2)$$

If we were to simplify, we get:

$$f(x) = \frac{3x}{\frac{2}{x} - 1} \quad (3)$$

$$= \frac{x}{x} \cdot \frac{3x}{\frac{2}{x} - 1} \quad (4)$$

$$= \frac{3x^2}{2 - x} \quad (5)$$

And from this we can conclude that the *limit* as  $x$  approaches 0 is 0. However, the simplification step involved multiplying by  $\frac{x}{x}$  which is undefined for  $x = 0$ . The function, as it was originally written, must exclude 0 from its domain. This is plotted in Fig. 1.

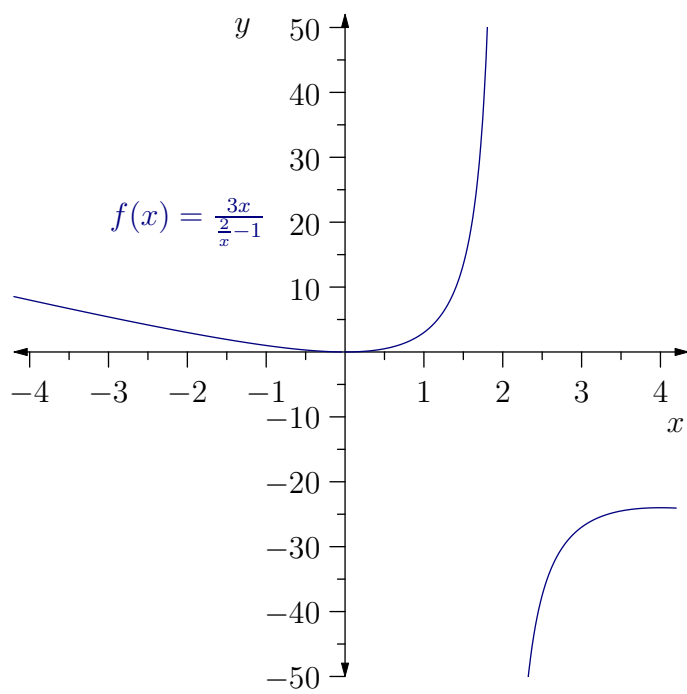


Figure 1: Graph of the function  $f$

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