Domains of Functions - Example 6

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Consider the expression:

$$f(x) = \frac{3x}{\frac{2}{x} - 1} \tag{1}$$

For what real numbers is this well-defined? The only thing to look out for is division-by-zero. The denominator is $2/x - 1$, so if we are to avoid division-by-zero we must exclude $x = 2$. The expression $2/x$ also contains a division, and so we must also exclude $x = 0$. In set theory notation, we can write the domain as:

$$D = (-\infty, 0) \cup (0, 2) \cup (2, \infty) \tag{2}$$

If we were to simplify, we get:

$$f(x) = \frac{3x}{\frac{2}{x} - 1} \tag{3}$$

$$= \frac{x}{x} \cdot \frac{3x}{\frac{2}{x} - 1} \tag{4}$$

$$= \frac{3x^2}{2 - x} \tag{5}$$

And from this we can conclude that the *limit* as x approaches 0 is 0. However, the simplification step involved multiplying by $\frac{x}{x}$ which is undefined for $x = 0$. The function, as it was originally written, must exclude 0 from it's domain. This is plotted in Fig. 1.

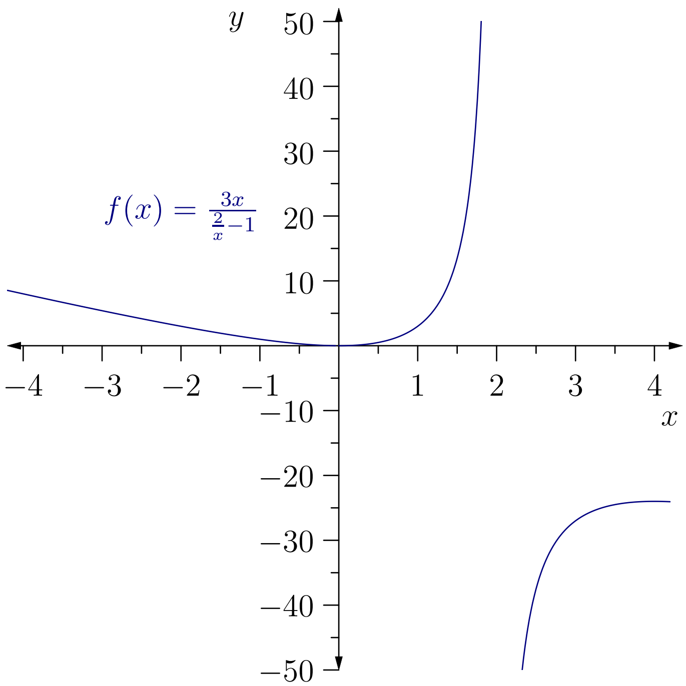

Figure 1: Graph of the function f