Domains of Functions - Example 7

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Consider the expression:

$$f(x) = \frac{1}{\frac{1}{x} - 1} \tag{1}$$

We can simplify this by the following:

$$\begin{aligned} f(x) &= \frac{1}{\frac{1}{x} - 1} && (2) \\ &= \frac{x}{x} \cdot \frac{1}{\frac{1}{x} - 1} && (3) \\ &= \frac{x}{1 - x} && (4) \end{aligned}$$

If we examine this new expression we might conclude that the only value where $f(x)$ may be undefined is $x = 1$, since then $1 - x$ would evaluate to zero and our expression would have a division-by-zero. However, in the simplification we multiplied our expression by $\frac{x}{x}$. For all values other than zero this simplifies to 1, and is therefore a valid step, but for 0 this expression is undefined. Looking back to our original expression we see that $\frac{1}{x}$ occurs in the denominator. For this to be well-defined, we need to avoid $x = 0$ as well. The domain can be written as:

$$D = (-\infty, 0) \cup (0, 1) \cup (1, \infty) \tag{5}$$

This function is plotted in Fig. 1.

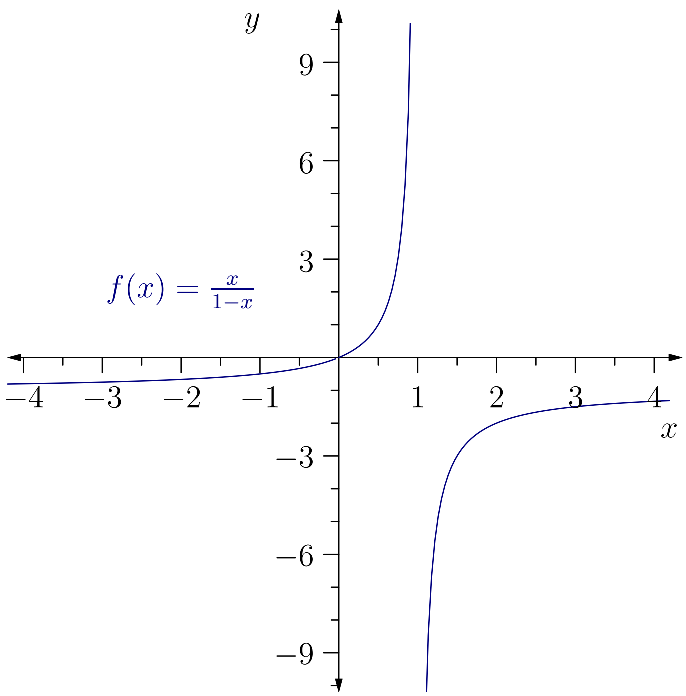

Figure 1: Graph of the function f