

# Tangent Lines - Example 2

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Let's compute the tangent line of  $f(x) = x^2$  at  $x_0 = 0.2$ . The difference quotient for any real number  $x \in \mathbb{R}$  is:

$$\frac{f(x+h) - f(x)}{h} \tag{1}$$

For our function  $f(x) = x^2$  we get:

$$\frac{(x+h)^2 - x^2}{h} \tag{2}$$

This gives us the slope of the *secant* line between the points  $(x, f(x))$  and  $(x+h, f(x+h))$ . This is shown in Fig. 1. For small values of  $h$  the secant line approximates the tangent line, and the *limit* as  $h$  tends to zero is precisely the tangent line. The limit of the difference quotient is also the definition of the derivative of  $f$  at  $x$ :

$$\frac{df}{dx}(x) = f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \tag{3}$$

Let's explicitly evaluate the derivative of our function  $f(x) = x^2$  for any real number  $x \in \mathbb{R}$ . We'll then use this to calculate the equation of the tangent line. We have:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \tag{4}$$

$$= \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} \tag{5}$$

$$= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x^2}{h} \tag{6}$$

$$= \lim_{h \rightarrow 0} \frac{2xh + h^2}{h} \tag{7}$$

$$= \lim_{h \rightarrow 0} (2x + h) \tag{8}$$

$$= 2x \tag{9}$$

So, we have  $f'(x) = 2x$ . Let's use this. The slope of the tangent line at the point  $x_0$  is given by  $f'(x_0)$ . We've chosen  $x_0 = 0.2$ , so we have  $f'(x_0) = f'(0.2) =$

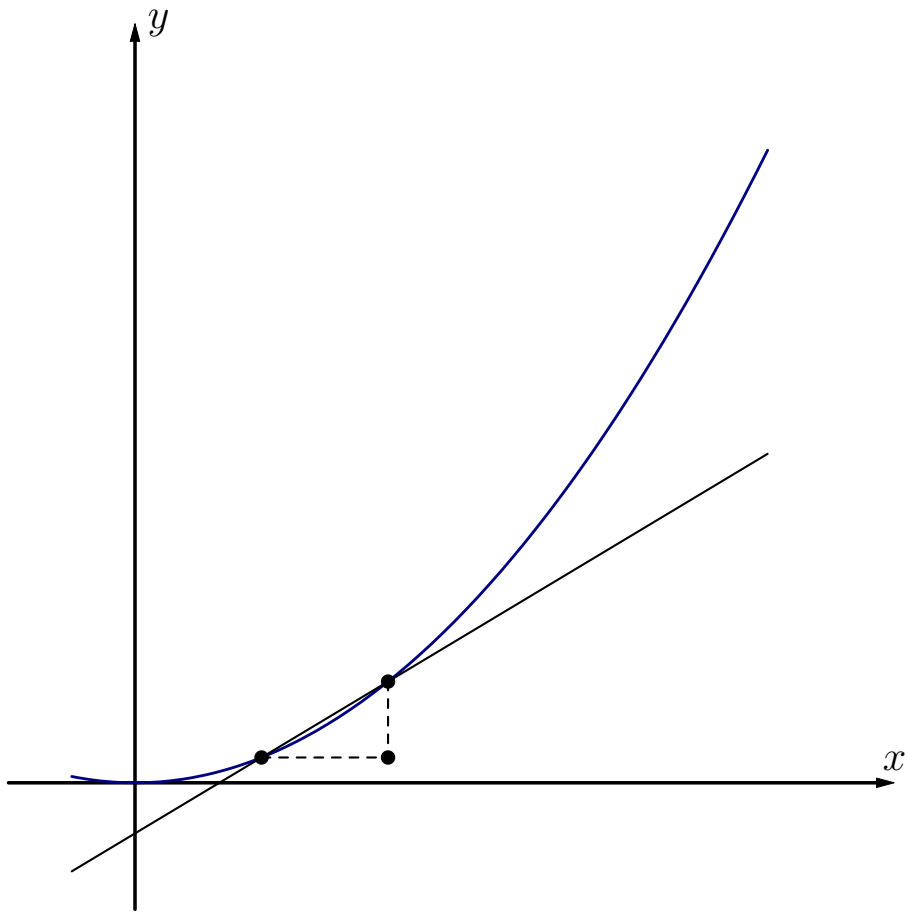


Figure 1: Secant Line for  $f$

$2(0.2) = 0.4$ . That is, the slope of the tangent line is 0.4. The tangent line has the formula:

$$y_T = f'(x_0)(x - x_0) + y_0 \quad (10)$$

Where  $y_0 = f(x_0)$ . For  $x_0 = 0.2$ , we have  $y_0 = 0.04$ . So the tangent line is:

$$y_T = 0.2(x - x_0) + 0.04 \quad (11)$$

This is plotted in Fig. 2.

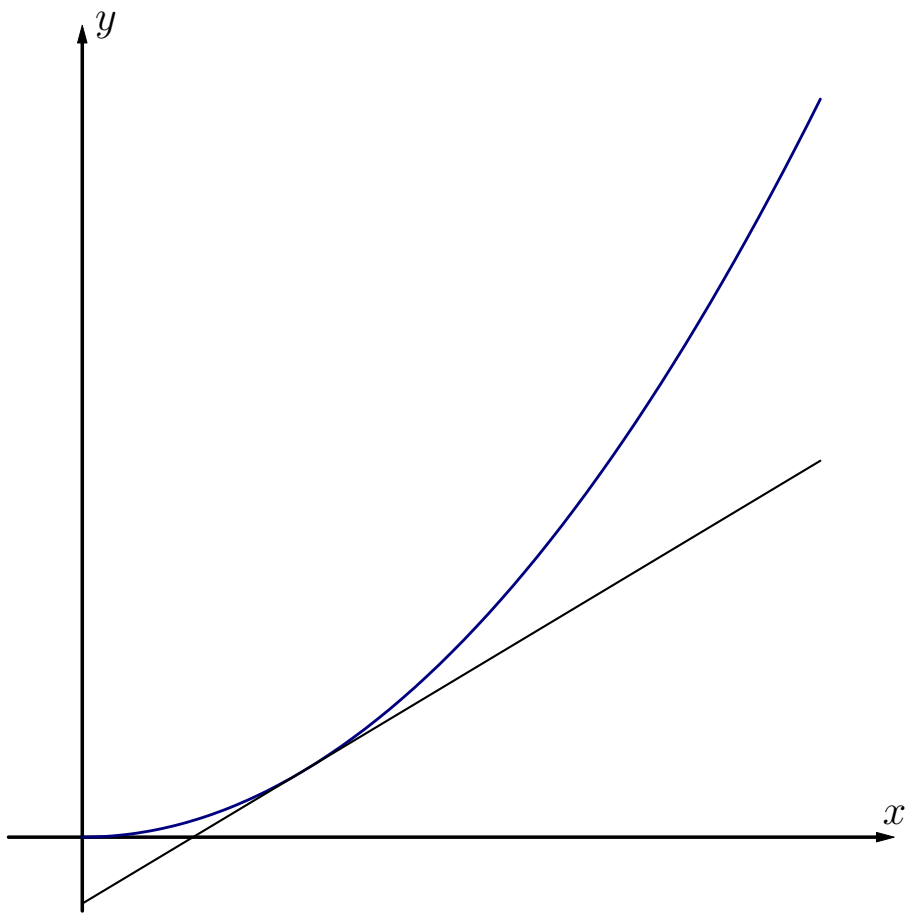


Figure 2: Tangent Line for  $f$

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