# Tangent Lines - Example 2

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Let's compute the tangent line of $f(x) = x^2$ at $x_0 = 0.2$. The difference quotient for any real number $x \in \mathbb{R}$ is:

$$\frac{f(x+h) - f(x)}{h} \tag{1}$$

For our function $f(x) = x^2$ we get:

$$\frac{(x+h)^2 - x^2}{h} \tag{2}$$

This gives us the slope of the *secant* line between the points $(x, f(x))$ and $(x+h, f(x+h))$. This is shown in Fig. 1. For small values of $h$ the secant line approximates the tangent line, and the *limit* as $h$ tends to zero is precisely the tangent line. The limit of the difference quotient is also the definition of the derivative of $f$ at $x$:

$$\frac{\mathrm{d}f}{\mathrm{d}x}(x) = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \tag{3}$$

Let's explicitly evaluate the derivative of our function $f(x) = x^2$ for any real number $x \in \mathbb{R}$. We'll then use this to calculate the equation of the tangent line. We have:

$$\begin{aligned}
f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} && (4)\\
&= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} && (5)\\
&= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} && (6)\\
&= \lim_{h \to 0} \frac{2xh + h^2}{h} && (7)\\
&= \lim_{h \to 0} (2x + h) && (8)\\
&= 2x && (9)
\end{aligned}$$

So, we have $f'(x) = 2x$. Let's use this. The slope of the tangent line at the point $x_0$ is given by $f'(x_0)$. We've chosen $x_0 = 0.2$, so we have $f'(x_0) = f'(0.2) =$

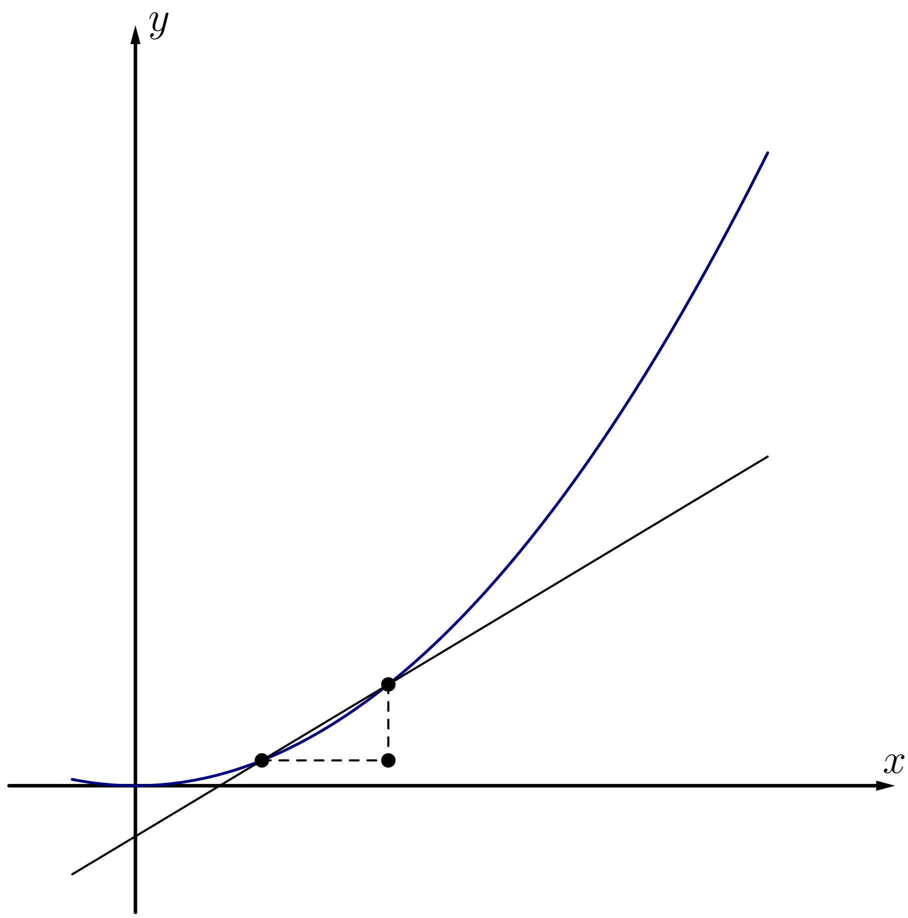


Figure 1: Secant Line for $f$

$2(0.2) = 0.4$. That is, the slope of the tangent line is 0.4. The tangent line has the formula:

$$y_T = f'(x_0)(x - x_0) + y_0 \tag{10}$$

Where $y_0 = f(x_0)$. For $x_0 = 0.2$, we have $y_0 = 0.04$. So the tangent line is:

$$y_T = 0.2(x - x_0) + 0.04 \tag{11}$$

This is plotted in Fig. 2.

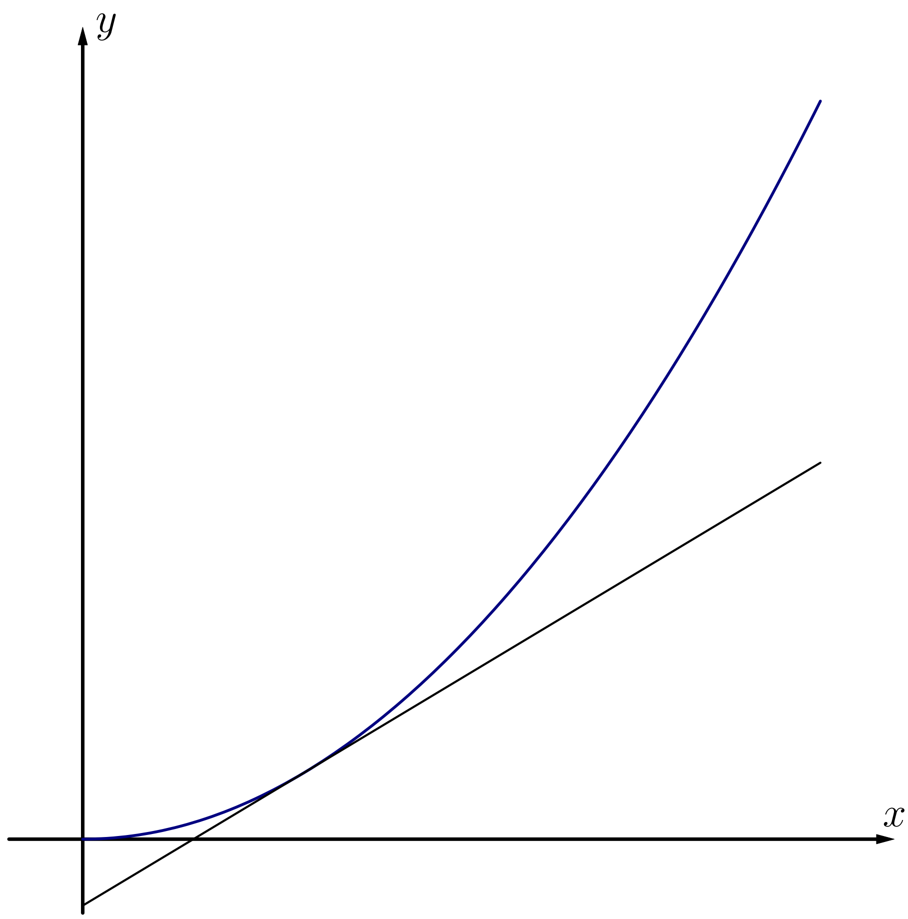


Figure 2: Tangent Line for $f$