Point-Set Topology

Math 54 (Summer 2023)

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Class Time: MWF 12:50 - 1:55, Kemeny 108
Office Hours: MWF 2:00 - 3:00, Kemeny 241
x-Hour: T 1:20 - 2:10, Kemeny 108

Course Description

Topology is the study of *generalized geometry*. In geometry one considers rigid motions that leave an object in the same shape such as translations, rotations, and reflections. Topology allows you to *continuously deform* the object by stretching, twisting, squeezing, and bending. Because of this it is often said a topologist can't tell the difference between a coffee cup and a donut (they're both objects with one hole in it). Certain geometrical properties like distance, volume, and curvature are not preserved by the motions allowed in topology, meaning other *topological invariants* are needed to discern different types of *topological spaces*. In this course we will discuss what topological spaces are, continuous functions, and various topological properties such as connectedness and compactness.

Course Objectives

By the end of the course one should have a solid understanding as to what topological spaces are, how they are used in other branches of mathematics and physics, and know about some of the important topological invariants that are used in the field. The course is very pictorial, and it is hoped students will appreciate topology for both its practical applications and also its aesthetics.

Prerequesites

Logically, there are no mathematical prerequesites. The course will cover the basics of logic and set theory before diving in to the main topics. Historically, set theory and point-set topology were originally the same field so it is fitting to start here. Practically, a student needs some mathematical maturity. Understanding the need for proof, and having some motivation for topology is recommended. This can be found in a course on real analysis (Math 35).

Textbook

There is **no required textbook**. I will be handing out notes that will cover everything needed in the class. For those who become really fascinated with topology, the following are *recommended*, but not required.

- *Topology*, second edition by James Munkres
- *Introduction to Topology*, second edition by Theodore W. Gamelin and Robert Everist Greene.
- *General Topology* by Stephen Willard
- *General Topology* by John L. Kelley

Grading

Your grade will consist of 3 components:

1. Bi-weekly homework assignments (50%)
2. Take-Home Midterm Exam (20%)
3. Take-Home Final Exam (30%)

The homework will consist of several problems broken into parts. For the most part they will not have the feel of a problem from a calculus course where you have a concept and you need to apply. Rather, you'll have to ponder over the ideas and perhaps *create* something new.

Order of Topics

There are 10 topics that we'll cover. Coincidentally there are 10 weeks in the course. However, some topics will get longer amounts of time than others.

1. Motivation, sets, and logic.
    - What *or*, *and*, and *if-then* mean.
    - The notion of unions, intersections, and complements.
    - Functions and cardinality.
    - The axiom of choice and well-ordering.
2. Metric spaces
    - Definition of metric spaces, the metric function.
    - Sequences, what it means to be open and closed.
    - Continuity, three equivalent definitions.
3. Topologicaly spaces
    - Definition of topological spaces.
    - Definition of continuous functions.
    - Definition of homeomorphism.

- The Hausdorff property.
- Equivalent notions of topological spaces.

4. Creating new spaces from old.
    - Subspaces.
    - Product spaces.
    - Quotient spaces.
    - Homotopy and homotopy equivalence.
5. Bases
    - Definition of bases, sub-bases.
    - Sequential spaces, their relation to metric spaces.
    - First and second countable spaces.
    - Separable spaces, what these terms mean for metric spaces.
6. Separation properties.
    - Regular spaces, normal spaces, and more.
7. Connectedness
    - Definition of connected, path connected.
    - What it means to be simply path connected.
8. Compactness
    - Definition, Tychonoff theorem.
    - Weaker notions (paracompactness, locally compact).
9. Metrization
    - Metrization theorems.
    - Topological properties of metrizable spaces.
10. Advanced Topics
    - Several possibilities here, pending time:
        - Manifolds and hyperbolic geometry.
        - Knots and links.
        - Planar topology and the Jordan curve theorem.

The Honor Principle

On written homework, you are encouraged to work together, and you may get help from others, but you must write up the answers yourself. If you are part of a group of students that produces an answer to a problem, you cannot then copy that group answer. You must write up the answer individually, in your own words.

On exams, you may not give or receive help from anyone. The take-home portion of the exams in this course are open book and open notes. But no other sources are allowed. In particular accessing the internet to research answers is an Honor Code violation. During take home exams, it is perfectly ok to interact with classmates as long as you do not discuss the exam in any way. Seriously, you shouldn't even comment on length, difficulty, or anything else.

Disabilities

Students with disabilities who may need disability-related academic adjustments and services for this course are encouraged to see me privately as early in the term as possible. Students requiring disability-related academic adjustments and services must consult the Student Accessibility Services office (Carson Hall, Suite 125, 646-9900). Once SAS has authorized services, students must show the originally signed SAS Services and Consent Form and/or a letter on SAS letterhead to their professor. As a first step, if students have questions about whether they qualify to receive academic adjustments and services, they should contact the SAS office. All inquiries and discussions will remain confidential.

Stress and Mental Well-Being

The academic environment at Dartmouth is challenging, our terms are intensive, and classes are not the only demanding part of your life. There are a number of resources available to you on campus to support your wellness, including your undergraduate dean, Counseling and Human Development, and the Student Wellness Center.

Religious Observances

Some students may wish to take part in religious observances that occur during this academic term. If you have a religious observance that conflicts with your participation in the course, please meet with me before the end of the second week of the term to discuss appropriate accommodations.