Math 38 - Graph theory

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Dartmouth College

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Introductions

- name
- class year
- major
- why are you taking this class
- fun fact about yourself

Overview of the course

Course website:
`https://canvas.dartmouth.edu/courses/65155`

- Textbook: Douglas B. West, *Introduction to Graph Theory*
- Assessment: weekly homework + 4 in-class quizes (check the dates!) + final exam + class participation
- Check the syllabus on the website

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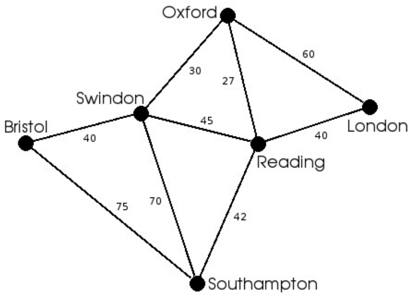
- Network flow problems. Used to optimize transmission of data, and to understand traffic patterns.
- Traveling Salesman Problem: In what order should a traveling salesman visit some cities to minimize the travel time?
- Map coloring: How many different colors do we need to color the regions of a map so that neighboring regions receive different colors?

Minimum Connector Problem

Find the most cost-effective way to connect some towns by building roads/railroads.



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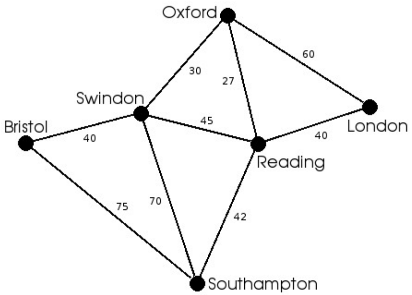

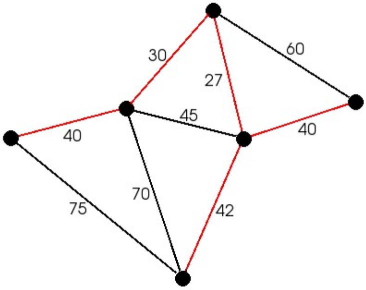

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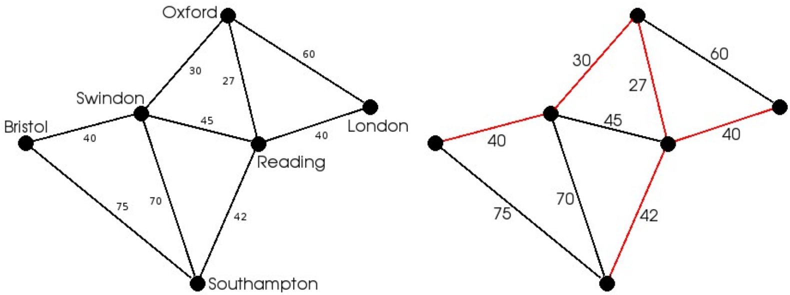

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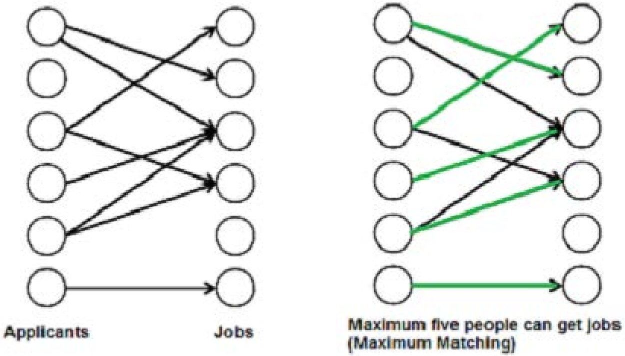
Similar problems involve connecting computers in a network by laying cable, or connecting houses to the electric grid.

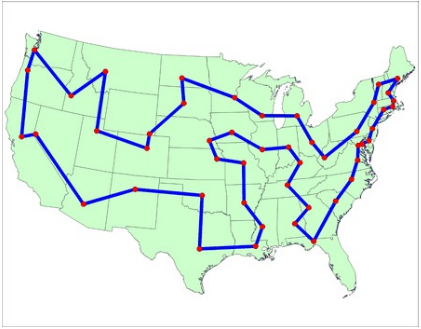
Finding matchings

Consider a set of applicants, each of whom is capable of doing some subset of jobs. Find an assignment of applicants to jobs such that each job is assigned to one capable applicant.



Traveling Salesman Problem

Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city and returns to the starting point?



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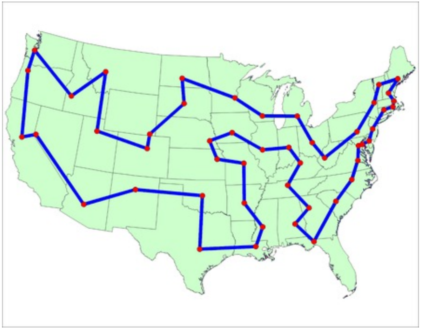

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Other related problems involve finding the most efficient way for a robot's arm to solder all the connections on a printed circuit board.

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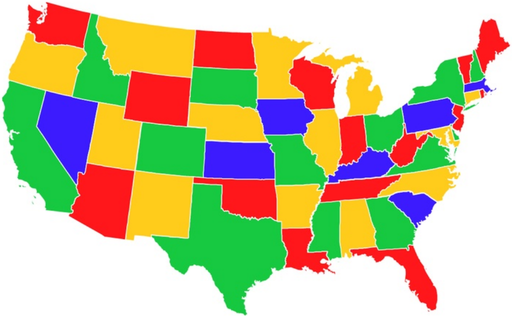

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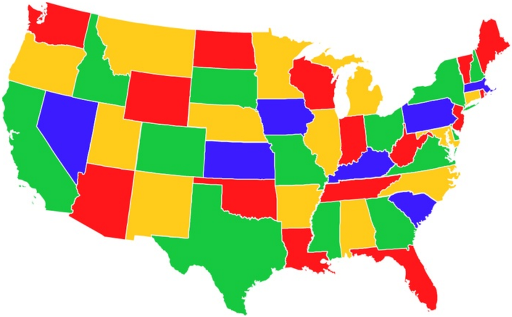

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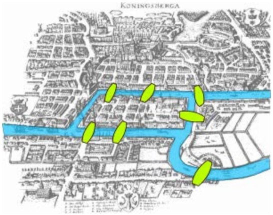
Can any map be colored using four colors?

Origins of Graph Theory: the Königsberg bridge problem



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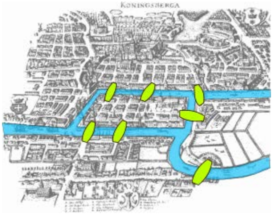

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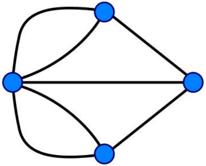

where the dots are land masses and the lines are bridges.

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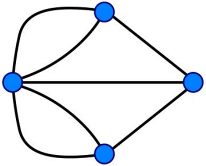

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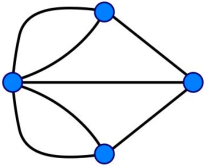

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Similar questions are relevant nowadays. The **Chinese Postman Problem** asks to design a route for a postman (or garbage truck, snowplow, etc.) around a city, so that every road is traversed at least once.
To minimize driving time, we would like each road to be traversed exactly once. Can this be done? Otherwise, what is the most efficient way?

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A **simple graph** is a graph without loops or multiple edges.