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Corollary

If G is bipartite with no isolated vertices, then

$$\alpha(G) = \beta'(G).$$

Chapter 4
Connectivity and paths

Connectivity

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In this chapter, graphs will have no loops (since loops do not affect connectivity).

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