Cut-edges and cut-vertices

Definition

A **cut-edge** or **cut-vertex** of a graph is an edge or vertex, respectively, whose deletion increases the number of components.

Cut-edges and cut-vertices

Definition

A **cut-edge** or **cut-vertex** of a graph is an edge or vertex, respectively, whose deletion increases the number of components.

Notation:

- $G - e$ is the graph obtained by deleting the edge e from G. Note that when we delete an edge we do not remove its endpoints.

Cut-edges and cut-vertices

Definition

A **cut-edge** or **cut-vertex** of a graph is an edge or vertex, respectively, whose deletion increases the number of components.

Notation:

- $G - e$ is the graph obtained by deleting the edge e from G. Note that when we delete an edge we do not remove its endpoints.
- $G - v$ is the graph obtained by deleting the vertex v from G. Note that when we delete a vertex we delete all the edges incident to it.

Cut-edges and cut-vertices

Definition

A cut-edge or cut-vertex of a graph is an edge or vertex, respectively, whose deletion increases the number of components.

Notation:

- $G - e$ is the graph obtained by deleting the edge e from G. Note that when we delete an edge we do not remove its endpoints.
- $G - v$ is the graph obtained by deleting the vertex v from G. Note that when we delete a vertex we delete all the edges incident to it.
- For $M \subseteq E(G)$, $G - M$ is the graph obtained by deleting the edges in the set M from G.

Cut-edges and cut-vertices

Definition

A cut-edge or cut-vertex of a graph is an edge or vertex, respectively, whose deletion increases the number of components.

Notation:

- $G - e$ is the graph obtained by deleting the edge e from G. Note that when we delete an edge we do not remove its endpoints.
- $G - v$ is the graph obtained by deleting the vertex v from G. Note that when we delete a vertex we delete all the edges incident to it.
- For $M \subseteq E(G)$, $G - M$ is the graph obtained by deleting the edges in the set M from G.
- For $S \subseteq V(G)$, $G - S$ is the graph obtained by deleting the vertices in the set S from G.

Induced subgraphs

Definition

An **induced subgraph** is a subgraph obtained by deleting a set of vertices.

Induced subgraphs

Definition

An **induced subgraph** is a subgraph obtained by deleting a set of vertices.

If $T \subseteq V(G)$ is the set of vertices that are left, we denote the corresponding induced subgraph by $G[T]$.

Note that $G[T]$ is the graph with vertex set T containing all the edges in G with both endpoints in T.

Induced subgraphs

Definition

An **induced subgraph** is a subgraph obtained by deleting a set of vertices.

If $T \subseteq V(G)$ is the set of vertices that are left, we denote the corresponding induced subgraph by $G[T]$.

Note that $G[T]$ is the graph with vertex set T containing all the edges in G with both endpoints in T.

Letting $\overline{T} = V(G) \setminus T$, we have that $G[T] = G - \overline{T}$.

Characterization of cut-edges

Theorem

An edge is a cut-edge if and only if it belongs to no cycle.

Characterization of bipartite graphs

We will characterize bipartite graphs in terms of their cycles.

Lemma

Every closed odd walk contains an odd cycle.

Characterization of bipartite graphs

We will characterize bipartite graphs in terms of their cycles.

Lemma

Every closed odd walk contains an odd cycle.

Note that this is not true if we replace *odd* with *even*.

Characterization of bipartite graphs

We will characterize bipartite graphs in terms of their cycles.

Lemma

Every closed odd walk contains an odd cycle.

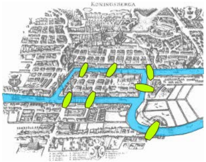
Note that this is not true if we replace *odd* with *even*.

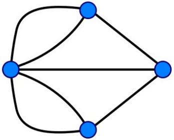
Theorem (König 1936)

A graph is bipartite if and only if it contains no odd cycle.

Characterization of Eulerian graphs

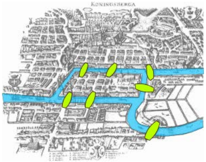
Recall the Königsberg bridge problem.

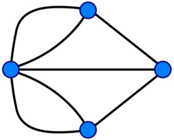




Characterization of Eulerian graphs

Recall the Königsberg bridge problem.



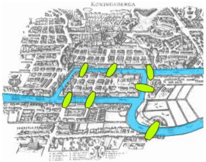


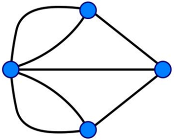
Definition

A graph is **Eulerian** if it has a closed walk that contains each edge exactly once. Such a walk is called an **Eulerian circuit**.

Characterization of Eulerian graphs

Recall the Königsberg bridge problem.





Definition

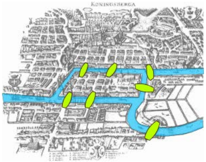
A graph is **Eulerian** if it has a closed walk that contains each edge exactly once. Such a walk is called an **Eulerian circuit**.

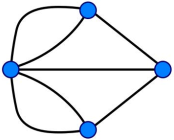
Lemma

If every vertex of a graph has degree ≥ 2, then it contains a cycle.

Characterization of Eulerian graphs

Recall the Königsberg bridge problem.





Definition

A graph is **Eulerian** if it has a closed walk that contains each edge exactly once. Such a walk is called an **Eulerian circuit**.

Lemma

If every vertex of a graph has degree ≥ 2*, then it contains a cycle.*

Theorem (Characterization of Eulerian graphs)

A connected graph is Eulerian if and only if all its vertices have even degree.