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Theorem (Mantel 1907)

The maximum number of edges in a triangle-free simple graph with n vertices is $\lfloor n^2/4 \rfloor$.

Degree sequences

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Proposition

A weakly decreasing sequence $(a_1, a_2, \ldots, a_n)$ of non-negative integers is the degree sequence of some graph if and only if $a_1 + a_2 + \cdots + a_n$ is even.

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Theorem (Havel'55, Hakimi'62)

A weakly decreasing sequence $(d_1, d_2, \ldots, d_n)$ *is graphic if and only if the sequence*

$$(d_2 - 1, d_3 - 1, \ldots, d_{d_1} - 1, d_{d_1+1} - 1, d_{d_1+2}, \ldots, d_n)$$

is graphic.

Note that this sequence is obtained by deleting the largest element d_1 and subtracting 1 from the next d_1 largest elements.

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Definition

The underlying graph of a digraph D is the graph obtained by treating the edges of D as unordered pairs.