Instructor: Sarah Frei
Office: 426 Herman Brown Hall (HBH)
Email: sarah.frei@rice.edu
Class Meetings: 9 - 9:50am, MWF, 453 HBH

Office Hours:
Mon 2-3pm,
Fri 12-1pm,
or by appointment

Teaching Assistants: Nicholas Rouse, Office hour: Tues 1-2pm.

Prerequisites: Math 463. Familiarity with Gröbner bases will be assumed.

Textbook: *Introduction to Commutative Algebra*, Atiyah & Macdonald. ISBN: 978-0201407518. Other suggested references include: *Commutative Algebra with a view Toward Algebraic Geometry*, Eisenbud; *Undergraduate Commutative Algebra*, Reid; *Algebra*, Lang; *Abstract Algebra*, Dummit & Foote.

Learning Outcomes: A successful student in this course should:

- Be able to state, understand, and apply structural theorems from commutative algebra to solve problems of a theoretical nature. Examples of these theorems include the Yoneda Lemma, the Cayley-Hamilton theorem, Nakayama's Lemma, the Snake and 5- Lemmas, local properties, primary decomposition, Noether normalization, Hilbert's Nullstellensatz, and the Going-up Theorem, among others.
- Understand basic concepts in category theory, including universal objects and properties, in order to streamline structural proofs and the solution of concrete exercises in commutative algebra.
- Understand the equivalence between the category of commutative rings with unit, and the category of affine schemes, setting up the stage for modern algebraic geometry.
- Understand how abstract algebra underpins some other areas of mathematics, e.g., how exterior algebras give a convenient language for differential forms, how projective modules over a ring can give rise to vector bundles, how to properly define a determinant, etc.

Homework: Due once a week, on **Wednesday, at 5pm** in my office. You are welcome to hand in the homework at the beginning of lecture on Wednesday.

The homework is not pledged and you can collaborate with other students in the class. In fact, you are very much *encouraged* to do so. However, you are not allowed to look up solutions in any written form; in particular, you are not allowed to look up solutions online. Students caught violating this rule will be reported to the Honor Council. You should write up your solutions individually.

Undergraduates enrolled in Math 464 will have a reduced homework load. Mathematics graduate students should enroll in Math 564.

Homework is a very important component of the course. This class has a heavy workload, and you should expect to spend a lot of time doing homework. Math 464/564 is in many ways similar to a language course: you must get lots of hands-on practice to internalize the definitions.

Exams: There will be a take-home midterm exam, due the week of **February 24th**. There will also be a 5-hour take-home final exam, due during Exam period no earlier than the Registrar's Office scheduled final exam for the course, as per University Policy.

The date for the final exam is set by the Registrars office and is not available at this time. It is the policy of the Mathematics Department that no final may be given early to accommodate student travel plans. If you make travel plans that later turn out to conflict with the scheduled exam, then it is your responsibility to either reschedule your travel plans or take a zero on the final.

If an exam conflicts with a holiday you observe, please let me know.

Grading: Homework will count for 50% of your final grade. The midterm will count for 20% of your grade and the final exam will count for 30% of your grade.

Attendance: Attendance is not required. However, you are responsible for all the material and announcements covered in lecture. While Canvas is a valuable resource, not all announcements will be posted there. Nevertheless, you are responsible for reading any emails I send to the class through Canvas.

Expectations: In my experience as a student, most people do not follow all the details of a Math lecture in real time. During lecture, you should expect to witness the big picture of what's going on. You should pay attention to the lecturer's advice on what is important and what isn't. A lecturer spends a long time thinking about how to deliver a presentation of an immense amount of material; they do not expect you to follow every step, but they do expect you to go home and fill in the gaps in your understanding. Not attending lecture really hurts your chances at a deep understanding of the material.

Rice Honor Code: In this course, all students will be held to the standards of the Rice Honor Code, a code that you pledged to honor when you matriculated at this institution. If you are unfamiliar with the details of this code and how it is administered, you should consult the Honor System Handbook at `http://honor.rice.edu/honor-system-handbook/`. This handbook outlines the University's expectations for the integrity of your academic work, the procedures for resolving alleged violations of those expectations, and the rights and responsibilities of students and faculty members throughout the process.

Disability Resource Center: If you have a documented disability or other condition that may affect academic performance you should: 1) make sure this documentation is on file with the Disability Resource Center (Allen Center, Room 111 /`adarice@rice.edu` / x5841) to determine the accommodations you need; and 2) talk with me to discuss your accommodation needs.

Sexual Violence: Any student who has experienced sexual assault, relationship or dating violence, stalking, and/or sexual harassment may seek resources and help at safe.rice.edu. If you need immediate assistance, call RUPD/REMS at 713-348-6000. If you need to talk to someone, call 713-348-3311.

As a Responsible Employee, if I learn of sexual or relationship violence and/or harassment, I am required to report it to one of the Title IX coordinators.

Topics to be covered

I plan to cover most of Chapters 1-8 of Atiyah & Macdonald, supplemented by material on category theory, and computational techniques developed in the 50 years(!) since Atiyah & Macdonald was first published.

1. **Rudiments of Category theory:** Definitions, examples. Equivalences of categories. Adjoint functors. Representable functors and the Yoneda Lemma. Products, coproducts, fibered products. Universal properties. Limits.

2. **Commutative Algebra:** Some ideal theory beyond Math 463. Tensor and exterior algebra. Flat, free and projective modules. Localization. Gröbner Bases. Integrality. Chain conditions. Noetherian and Artinian rings. Dedekind domains.

3. **Further Galois Theory:** Inverse limits; infinite Galois theory.

4. **Rudimentary Algebraic Geometry:** The category of affine schemes. Hilbert Nullstellensatz.

5. **Homological Algebra:** Complexes; homology. Injective modules and derived functors. Ext and Tor. Time permitting: basics of derived categories.