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Abstracts[1]

Tayeb Aissiou•. *Semiclassical limits of eigenfunctions of the Laplacian on $\mathbb{T}^n$.*

We will present a proof of the conjecture formulated by D. Jakobson in 1995, which states that on a (n)-dimensional flat torus $\mathbb{T}^n$, the Fourier series of squares of the eigenfunctions $|\phi_\lambda|^2$ of the Laplacian have uniform l^n bounds that do not depend on the eigenvalue λ. The proof is a generalization of the argument presented in papers [1,2] and requires a geometric lemma that bounds the number of codimension one simplices which satisfy a certain restriction on an n-dimensional sphere $S^n(\lambda)$ of radius $\sqrt{\lambda}$. We will present a sketch of the proof of the lemma.

Pierre Albin*. *Ricci flow and the determinant of the Laplacian on non-compact surfaces.*

The determinant of the Laplacian is an important invariant of closed surfaces and has connections to the dynamics of geodesics, Ricci flow, and physics. Its definition is somewhat intricate as the Laplacian has infinitely many eigenvalues. I'll explain how to extend the determinant of the Laplacian to non-compact surfaces where one has to deal with additional difficulties like continuous spectrum and divergence of the trace of the heat kernel. On surfaces (even non-compact) this determinant has a simple variation when the metric varies conformally. I'll explain how to use Ricci flow to see that the largest value of the determinant occurs at constant curvature metrics.This is joint work with Clara Aldana and Frederic Rochon.

Nalini Anantharaman*. *Semiclassical measures for the Schrödinger equation on the torus (joint work with F.Macia).*

Our main result is the following : let (u_n) be a sequence in $L^2(\mathbb{T}^d)$, such that $\|u_n\|_{L^2(\mathbb{T}^d)} = 1$ for all n. Consider the sequence of probability measures ν_n on $\mathbb{T}^d$, defined by $\nu_n(dx) = (\int_0^1 |e^{it\Delta/2}u_n(x)|^2 dt)dx$. Let ν be any weak limit of the sequence (ν_n): then ν is absolutely continuous. This generalizes a former result of Bourgain and Jakobson, who considered the case when the functions u_n are eigenfunctions of the Laplacian. Our approach is different from theirs, it relies on the notion of (two-microlocal) semiclassical measures, and the properties of the geodesic flow on the torus.

Teresa Arias-Marco•. *On inaudible curvature properties of closed Riemannian manifolds.*

Following Mark Kac, it is said that a geometric property of a compact Riemannian manifold can be heard if it can be determined from the eigenvalue spectrum of the associated Laplace operator on functions. On the contrary, D'Atri spaces, manifolds of type A,

[1]Speakers are marked with an asterisk; poster presenters with a bullet.

probabilistic commutative spaces, C-spaces, TC-spaces, and GC-spaces have been studied by many authors as symmetric-like Riemannian manifolds. In this article, we prove that for closed Riemannian manifolds, none of the properties just mentioned can be heard. Another class of interest is the class of weakly symmetric manifolds. We consider the local version of this property and show that weak local symmetry is another inaudible property of Riemannian manifolds. This is joint word with Dorothee Schueth.

Gregory Berkolaiko*. *Nodal domains and spectral critical partitions on graphs*

The k-th eigenfunction of a Schrodinger operator on a bounded regular domain Ω with Dirichlet boundary conditions defines a partition of Ω into n nodal subdomains. A famous result by Courant establishes that $n \leq k$; the number $k - n$ will be referred to as the nodal deficiency.

The nodal subdomains, when endowed with Dirichlet boundary conditions, have equal first eigenvalue, which coincides with the k-th eigenvalue of the original Schrodinger problem. Additionally, the partition is bipartite, i.e. it consists of positive and negative subdomains (corresponding to the sign of the eigenfunction), with two domains of the same sign not sharing a boundary.

Conversely, for a given partition, define the energy of the partition to be the largest of the first Dirichlet eigenvalues of its subdomains. An n-partition with the minimal energy is called the minimal n-partition. It is interesting to relate the extremal properties of the partitions to the eigenstates of the operator on Ω. Recently, Helffer, Hoffmann-Ostenhof and Terracini proved that n-th minimal partition is bipartite if and only if it corresponds to a Courant-sharp eigenfunction (an eigenfunction with nodal deficiency zero).

We study partitions on quantum graphs and discover a complete characterization of eigenfunctions as critical equipartitions. More precisely, equipartitions are partitions with all first eigenvalues equal. We parameterize the manifold of all equipartitions and consider the energy of an n-equipartition as a function on this manifold. For a generic graph and large enough n we establish the following theorem: a critical point of the energy function with b unstable directions is a bipartite equipartition if and only if it corresponds to an eigenfunction with nodal deficiency b. Since by constructions it has n nodal domains it is therefore the $n + b$-th eigenfunction in the spectral sequence.

Since at a minimum the number of unstable directions is $b = 0$, our results include the quantum graph analogue of the results of Helffer et al. They also provide a new formulation of known bounds on the number of nodal domains on generic graphs.

This is joint work with Rami Band, Hillel Raz and Uzy Smilansky.

Shimon Brooks. *Wave Interferences, Positive Entropy, and QUE.*

In joint work with E. Lindenstrauss, we establish QUE in two settings - joint eigenfunctions of the Laplacian and one Hecke operator on compact congruence surfaces, and joint eigenfunctions of the two partial Laplacians on compact quotients of $\{H\} \times \{H\}$ - by showing positive entropy on a.e. ergodic component of limit measures. The proof hinges on a recipe for dispersing successive waves in such a way that they interfere desirably.

Semen Dyatlov•. *Quasi-Normal modes for Kerr-de Sitter black holes.*

We present a rigorous definition of quasi-normal modes for Kerr-de Sitter black holes and show that, if the rotation parameter is sufficiently small, these modes do not have any accumulation poles. We also explain the connection with the wave equation.

Stephen Fulling*. *Index theorems for quantum graphs.*

The analog of the index theorem for manifolds has been constructed in several ways for the Laplacian on a quantum graph. The Laplacian factors as the product of two first-order operators, and hence the graph's structure, spectrum, and heat kernel are all related. This research in collaboration with P. Kuchment and J. H. Wilson has been published in J. Phys. A 40 (2007) 14165-14180.

Second title and abstract: *Vacuum Stress and Closed Paths in Rectangle, Pistons, and Pistols.* S. A. Fulling, L. Kaplan, K. Kirsten, Z. H. Liu, K. A. Milton. Journal of Physics A 42 (2009), 155402.

Rectangular cavities are solvable models that nevertheless touch on many of the controversial or mysterious aspects of the vacuum energy of quantum fields. This paper is a thorough study of the two-dimensional scalar field in a rectangle by the method of images, or closed classical (or optical) paths, which is exact in this case. For each point r and each specularly reflecting path beginning and ending at r, we provide formulas for all components of the stress tensor $T_{\mu\nu}(r)$, for all values of the curvature coupling constant ξ and all values of an ultraviolet cutoff parameter. Arbitrary combinations of Dirichlet and Neumann conditions on the four sides can be treated. The total energy is also investigated, path by path. These results are used in an attempt to clarify the physical reality of the repulsive (outward) force on the sides of the box predicted by calculations that neglect both boundary divergences and the exterior of the box. Previous authors have studied "piston" geometries that avoid these problems and have found the force to be attractive. We consider a "pistol" geometry that comes closer to the original problem of a box with a movable lid. We find again an attractive force, although its origin and detailed behavior are somewhat different from the piston case. However, the pistol (and the piston) model can be criticized for extending idealized boundary conditions into short distances where they are physically implausible. Therefore, it is of interest to see whether leaving the ultraviolet cutoff finite yields results that are more plausible. We then find that the force depends strongly on a geometrical parameter; it can be made repulsive, but only by forcing that parameter into the regime where the model is least convincing physically.

Victor Guillemin*. *Heat trace invariants for group actions on Riemannian orbifolds.*

In the first of these two lectures I will discuss an extension to orbifolds of a classical theorem of Donnelly on heat trace invariants for isometries of Riemannian manifolds. (This extension is based on a very beautiful recent result of Dryden, Gordon, Greenwald, and Webb on heat trace asymptotics for the Laplace operator on orbifolds.) In the second lecture I'll discuss some applications of this result to inverse spectral problems: Define the "equivariant spectrum" of a compact Riemannian orbifold, X, to be the eigenvalues of the Laplace operator plus the representations of the isometry group of X on the associated eigenspaces. To what extent does this data determine X and the action on X of the isometry group? An interesting test case for this question is Kaehler metrics on toric orbifolds, and I'll report on some work in progress, joint with Emily Dryden and Rosa Sena-Dias, involving this example.

Andrew Hassell*. *Neumann eigenfunctions and a "method of particular solutions" for computing them.*

Recently, Alex Barnett (Dartmouth) and I made a precise analysis of the "method of particular solutions" for computing Dirichlet eigenfunctions and eigenvalues on a bounded domain of $\mathbb{R}^n$ with smooth boundary. In particular, we gave sharp upper and lower bounds on the accuracy of this method (at high energies) using very elementary techniques. In this talk, after reviewing the Dirichlet case, I will outline an analogous method for computing Neumann eigenfunctions and eigenvalues, which has the same accuracy at high energy. The analysis here is much more complicated, however, and relies on several new estimates on Neumann eigenfunctions at the boundary, the proofs of which require more sophisticated microlocal techniques. This work is also joint with Barnett.

Luc Hillairet*. *Generic simplicity using asymptotic separation of variables.*

We study the spectrum of an analytic family of quadratic forms that is asymptotic to a reference family of quadratic forms for which separation of variables applies. Using control estimates of semiclassical nature, we prove that the spectrum of the perturbed family inherits the generic simplicity of the unperturbed one. We use this to prove generic simplicity of several geometric situations including Euclidean triangles and hyperbolic triangles with one ideal vertex. This is joint work with Chris Judge.

Dmitry Jakobson*. *Gauss curvature of random metrics.*

We study Gauss curvature for random Riemannian metrics on a compact surface, lying in a fixed conformal class; our questions are motivated by comparison geometry. We explain how to estimate the probability that Gauss curvature will change sign after a random conformal perturbation of a metric, and discuss some extremal problems for that probability, and their relation to other extremal problems in spectral geometry.

If time permits, analogous questions will be considered for the scalar curvature in dimension $n > 2$, as well as other related problems (e.g. Q-curvature in even dimensions).

This is joint work with Y. Canzani and I. Wigman

Chris Judge. *Spectral simplicity and asymptotic separation of variables.*

See `http://arxiv.org/abs/1001.0235`. This is joint work with Luc Hillairet in which we develop a new method for proving generic simplicity of the Laplace spectrum in finite dimensional parameter spaces. For example, we prove that the generic triangle has simple Laplace spectrum. See the talk by Luc Hillairet entitled "Generic simplicity using asymptotic separation of variables" that will describe this joint work.

Thomas Kappeler*. *On properties of the spectrum of non selfadjoint Zakharov-Shabat operator.*

We describe generic properties of the periodic and Dirichlet spectrum of non selfadjoint Zakharov-Shabat operators and discuss applications to the focusing NLS equation. This is joint work with Philipp Lohrmann and Peter Topalov

Dubi Kelmer*. *A uniform spectral gap for congruence covers of a hyperbolic manifold*

An arithmetic hyperbolic manifold comes with a natural family of congruence covers. In many applications, it is useful to have a uniform spectral gap for this family. When the manifold itself comes from a congruence group there are very good bounds coming from known results towards the Ramanujan conjectures. In this talk, I will describe new results, based on an old method of Sarnak and Xue, giving an effective uniform gap for congruence covers of a (non-congruence) hyperbolic manifold.

Peter Kuchment*. *Parseval frames of exponentially decaying Wannier functions.*

When studying shift-invariant linear operators, the two basic sets of functions are very useful: plane waves and delta-functions, which are interchanged by the Fourier transform.

In case of equations invariant with respect to a lattice in $\mathbb{R}^n$ (e.g., the solid state's Schroedinger operator with periodic potential or photonic crystal theory's Maxwell operator in a periodic dielectric medium), the Fourier transform and the corresponding function sets are not useful anymore. However, analogs of these are well known: Floquet-Bloch transform replaces the Fourier one, Bloch functions play the role of plane waves, and the so called Wannier functions (associated with an isolated spectral band) are analogs of delta functions. All of these are heavily used in physics and mathematics. The problem arises with the localization of Wannier functions: one wants them to decay as fast as possible (exponentially is preferred). However, it has been known since 1980s (Thouless) that existence of an orthonormal basis of exponentially (in fact, even much slower) decaying Wannier functions in the subspace corresponding to the spectral band faces a topological obstruction in the form of possible non-triviality of a vector bundle on a torus. Efforts are still being devoted to finding cases where the bundle is trivial, although generically it is not.

The talk will contain a recent result showing that if one relaxes the orthonormal basis condition and asks only for a Parseval frame of such functions, it always exists. The topological obstruction determines how redundant the frame should be (if the bundle is trivial, the frame is a basis).

The result applies equally to a variety of periodic equations, including ones on abelian coverings of Riemannian manifolds, graphs, or quantum graphs.

Michael Levitin*. *Fourier transform, null varieties, and eigenvalues of Laplacian.*

We consider a quantity $\kappa(\Omega)$ – the distance to the origin from the null variety of the Fourier transform of the characteristic function of Ω. We conjecture, firstly, that $\kappa(\Omega)$ is maximized, among all convex balanced domains $\Omega \subset \mathbb{R}^d$ of a fixed volume, by a ball, and also that $\kappa(\Omega)$ is bounded above by the square root of the second Dirichlet eigenvalue of Ω. We prove some weaker versions of these conjectures in dimension two, as well as their validity for domains asymptotically close to a disk, and also discuss further links between $\kappa(\Omega)$ and the eigenvalues of the Laplacians, as well as relations to some unsolved classical problems of harmonic analysis.

Jeremy Marzuola*. *Eigenfunction concentration and Strichartz estimates for planar polygonal domains.*

We discuss the results of two separate projects studying properties of the Dirichlet or Neumann Laplacian on a planar domain. The first with Andrew Hassell and Luc Hillairet establishes a nonconcentration results for eigenfunctions of the Laplacian. The second with Matt Blair, G. Austin Ford and Sebastian Herr studies the boundedness of solutions to the Schroedinger equation on polygonal planar domains in space time norms. Both results rely heavily on treating the domain as a Euclidean Surface with Conic Singularities and the second result then draws heavily on work of Ford on the Euclidean Cone and the works of Burq-Gerard-Tzvetkov on smooth compact Riemannian manifolds.

Ben McReynolds. *Higher geometric spectra.*

This is recent work with Alan Reid on higher dimensional geometric spectra.

Roberto Miatello. *The spectral theory of the Atiyah-Patodi-Singer operator on compact flat manifolds (joint work with Ricardo Podesta).*

Let D be the Dirac-type boundary operator defined by Atiyah, Patodi and Singer, acting on smooth even forms of a compact flat Riemannian manifold M. We give an explicit formula for the multiplicities of the eigenvalues of D in terms of values of characters of exterior representations of $\mathrm{SO}(n)$. We obtain an expression for the eta series in terms of the Hurwitz zeta function, and as a consequence, a formula for the eta invariant. We compute it explicitly for some families, for instance in the case of cyclic holonomy groups of odd prime order. Also, we show there are large families of manifolds that are D-isospectral and non-homeomorphic to each other.

Djordje Milicevic. *Large values of eigenfunctions and arithmetic hyperbolic 3-manifolds of Maclachlan-Reid type.*

One aspect of quantum chaos are extreme values of high-energy eigenfunctions, which, on Riemannian manifolds of negative curvature, are not well understood and can depend heavily on the global geometry of the manifold. The principal result to be presented in this talk shows that there is a distinguished class of arithmetic hyperbolic 3-manifolds on which a sequence of L^2-normalized high-energy Hecke-Maass eigenforms achieve values as large as a power of the Laplacian eigenvalue. Power growth, which was first observed in this context by Rudnick and Sarnak, is (by far) not expected generically and stands in stark contrast with the statistical models suggested by the so-called random wave conjecture. Arithmetic hyperbolic 3-manifolds on which the exceptional behavior is exhibited are, up to commensurability, precisely those containing immersed totally geodesic surfaces, as described by Maclachlan and Reid. We also discuss the question of identifying the Hecke-Maass eigenforms achieving power growth.

Ori Parzanchevsky$^\bullet$ *Isospectrality and Representations of Finite Groups.*

The Sunada construction relies (or can be seen to rely) on the the trivial representations of finite subgroups in an object's group of isometries. In a recent work we explain how to obtain isospectrality from other representations of such groups, giving new examples of isospectral objects and shedding new light on existing ones. This is joint work with Rami Band.

Cyrus Peterpaul$^\bullet$ *Warped Products of Isospectral Graphs.*

Ralf Rueckriemen. *The Floquet spectrum of a quantum graph.*

We define the Floquet spectrum of a quantum graph as the collection of all spectra of operators of the form $D = (-i\frac{\partial}{\partial x} + \alpha(\frac{\partial}{\partial x}))^2$ where α is a closed 1-form. We show that the Floquet spectrum completely determines planar 3-connected graphs (without any genericity assumptions on the graph). It determines whether or not a graph is planar. Given the combinatorial graph, the Floquet spectrum uniquely determines all edge lengths of a quantum graph.

Peter Sarnak*. *The distribution of mass and zeros for high frequency eigenfunctions on the modular surface.*

After a brief introduction to the Quantum Unique Ergodicity Conjecture (QUE),we specialize to the basic arithmetic setting of the modular surface and Hecke eigenforms (both holomorphic and eigenfunctions of the Laplacian).We review the techniques that have led to the proof of the Conjecture in this case.In the second lecture we use related arithmetic techniques to investigate restriction properties to curves of such arithmetic eigenforms and use this together with related techniques to study the finer behaviour of the zeros(ie nodal lines in the Maass case) of such forms.

Dorothee Schueth*. *Local symmetry of harmonic spaces as determined by the spectra of small geodesic spheres.*

We show that in any harmonic space, the eigenvalue spectra of the Laplace operator on small geodesic spheres around a given point determine the norm of the covariant derivative of the Riemannian curvature tensor in that point. In particular, the spectra of small geodesic spheres in a harmonic space determine whether the space is locally symmetric. For the proof we use the first few heat invariants and consider certain coefficients in the radial power series expansions of the curvature invariants $|R|^2$ and $|Ric|^2$ of the geodesic spheres. Moreover, we obtain analogous results for geodesic balls with either Dirichlet or Neumann boundary conditions

Vladimir Sharafutdinov*. *Local audibility of a hyperbolic metric.*

A compact Riemannian manifold (M, g) is said to be locally audible if the following statement holds for every metric g' on M which is sufficiently close to g: if the metrics g and g' are isospectral then they are isometric. We prove local audibility of a compact locally symmetric Riemannian manifold of negative sectional curvature.

Mikhail Shubin*. *Solutions of non-linear equations in functions growing at infinity along the space variables.*

Most solvability classes of non-linear equations consist of functions which decay at infinity in appropriate classes of solutions. The first example of a different situation was given by A. Menikoff (1972), and treats the Korteweg-de Vries equation in classes of functions which are $O(|x|^\beta)$ as $|x| \to \infty$ with appropriate estimates of derivatives, and $\beta < 1$. Further examples were provided by C. Koning, G. Ponce, L. Vega (1977), I. Bondareva and M. Shubin (1982-1990), T. Kappeler, P. Perry, P. Topalov, M. Shubin (2008).

The main idea is constructing asymptotic power series which provide formal solutions of the problem, and use them as model solutions. This allows to reduce solving our equations in the classes of functions which decay at infinity arbitrarily fast.

Lior Silberman. *Higher-rank QUE.*

This work focuses on the equidistribution problem for automorphic forms on higher-rank symmetric spaces. There the classical dynamical system is a multiparameter flow with many symmetries (arising from number theory), giving an a-priori restrictions on the possible quantum limits. In earlier work with Venktesh it was seen that in some cases it is possible to show equidistribution of eigenstates which respect the symmetries (so-called "Hecke-Maass forms"); in recent work with Anantharaman it is shown that even if the eigenfunctions are not assumed to respect the symmetries, the limiting measure cannot be entirely singular to the uniform measure.

Elizabeth Stanhope. *Eigenvalues, eigenfunctions and Riemannian orbifolds.*

This work focuses on aspects of the Laplace operator on a compact Riemannian orbifold. We discuss ways to quantify the similarities between Laplace isospectral Riemannian orbifolds using metric geometry and critical point theory. We also use wave kernel methods to understand the behavior of eigenfunctions on a Riemannian orbifold.

Toshikazu Sunada*. *Quantum walks.*

This talk, including an introductory survey and a few results, treats quantum walks in view of "geometry of unitary operators". Ideas in discrete geometric analysis, developed to solve various problems in analysis on graphs, are effectively employed to give new insight to the theory which has been attracting computer scientists and physicists over the past decade. A limit theorem of large deviation type for the one dimensional lattice is stated to show a big discrepancy between quantum walks and classical walks.

Craig Sutton*. *Spectral isolation of bi-invariant metrics on compact Lie groups.*

This talk is motivated by the question of whether symmetric spaces (and other special classes of metrics) can be recognized via the spectrum. We show that a bi-invariant metric on a compact connected Lie group G is spectrally isolated within the class of left-invariant metrics. In fact, we prove that given a bi-invariant metric g_0 on G there is a positive integer N such that, within a neighborhood of g_0 in the class of left-invariant metrics of at most the same volume, g_0 is uniquely determined by the first N distinct non-zero eigenvalues of its Laplacian (ignoring multiplicities). In the case where G is simple, N can be chosen to be two. This is joint work with Carolyn Gordon and Dorothee Schueth.

Melissa Tacy*. *Eigenfunction L^p Estimates on Manifolds of Constant Negative Curvature.*

Estimates on the L^p norms of eigenfunctions (with eigenvalue λ^2) of the Laplace-Beltrami operator can be used to control the extent to which eigenfunctions can concentrate. In general such estimates are usually obtained by estimating operator norms of spectral clusters. The first major result in this direction is Sogge's Theorem on L^p norms for a spectral window of width one. In the case of a manifold without conjugate points Bérard was able to obtain a $\log \lambda$ improvement on eigenvalue multiplicity and therefore the same improvement on L^∞ eigenfunction estimates. His method was to shrink the spectral window under consideration by a factor of $\log \lambda$. Such shrinking requires long time control over the half wave propagator which is handled via the universal cover. Within this framework I will discuss L^p eigenfunction estimates for $p < \infty$ in the special case of a two dimensional manifold with constant negative curvature and obtain logarithmic improvements on Sogge's estimates. This work is joint with Andrew Hassell.

John Toth*. *Generic bounds for Laplace eigenfunctions.*

I will discuss recent generic supremum bounds for Laplace eigenfunctions of conformal families of Laplacians that are consistent with random wave predictions.

Zuoqin Wang. *Semi-classical Spectral Invariants of Schrödinger Operators.*

In joint work with V. Guillemin and A. Uribe, we describe how to compute the semi-classical spectral measures associated with the Schrödinger operators on $\mathbb{R}^n$, possibly with a magnetic potential or with a small perturbation, and, by examining the first few terms in the asymptotic expansion of this measure, obtain inverse spectral results in one and two dimensions.

Martin Weilandt. *Isospectral metrics on weighted projective spaces.*

We construct the first examples of families of bad Riemannian orbifolds which are isospectral with respect to the Laplacian but not isometric. In our case these are particular fixed weighted projective spaces equipped with isospectral metrics obtained by a generalization of Schüth's version of the torus method.

Nina White$^\bullet$. *Bounds on λ_1 for certain classes of hyperbolic manifolds.*

Buser and Cheeger showed that the following relationships hold between the Cheeger constant $h(M)$, and the smallest positive eigenvalue of the Laplace operator on $\lambda_1(M)$

$$\frac{h^2(M)}{4} \leq \lambda_1(M) \leq 2a(n-1)h(M) + 10h^2(M),$$

where M is n–dimensional and has Ricci curvature bounded below by $-(n-1)a^2$. In the case of hyperbolic three–manifolds fibering over the circle, Buser's inequality, the upper bound, can be restated as follows:

$$\lambda_1(M) \leq \frac{K}{\mathrm{vol}(M)},$$

where K depends on the genus of the fiber. In this poser, we sharpen this upper bound for ϵ–thick hyperbolic manifolds fibering over the circle and generalize this new upper bound to a larger class of hyperbolic manifolds using a theorem of Biringer and Souto.

Igor Wigman. *Fluctuations of the nodal length of random Laplace eigenfunctions.*

Using the multiplicities of the Laplace eigenspace on the sphere (the space of spherical harmonics) we endow the space with Gaussian probability measure. This induces a notion of random Gaussian spherical harmonics of degree n having Laplace eigenvalue $E = n(n+1)$. We study the length distribution of the nodal lines of random spherical harmonics.

It is known that the expected length is of order n. It is natural to conjecture that the variance should be of order n, due to the natural scaling. Our principal result is that, due to an unexpected cancelation, the variance of the nodal length of random spherical harmonics is of order $\log(n)$. This behaviour is consistent to the one predicted by Berry for nodal lines on chaotic billiards (Random Wave Model).

Seunghee Ye$^\bullet$. *Isospectral surfaces with distinct covering spectra*

The covering spectrum of a manifold measures size of one-dimensional holes in a Riemannian manifold. More precisely, if we are given a manifold M, we look at a certain family of covering spaces $\{\tilde{M}^\delta\}$ indexed by the positive real numbers such that $\tilde{M}^\delta$ covers $\tilde{M}^{\delta'}$ whenever $\delta < \delta'$. Then, the covering spectrum consists of the values of δ where we see a change or a "jump" in the isomorphism type of the covering spaces $\tilde{M}$. It is an interesting fact that these jumps are half the length of the shortest closed geodesic in certain free homotopy classes. Consequently, it is natural to ask whether the covering spectrum is a spectral invariant. Using a variant of Sunada's method, De Smit, Gornet, and Sutton showed that for compact manifolds of dimensions three or greater, the covering spectrum is not a spectral invariant. In a later article, they show that the covering spectrum is not a spectral invariant of surfaces by using a graph theoretic method. In this talk I will prove that the covering spectrum is not a spectral invariant of surfaces by using this graph theoretic method with new examples that are easier to understand than those used by De Smit, et al.

Steve Zelditch*. *Quantum ergodic restriction theorems.*

Quantum ergodicity involves the matrix elements $< A\phi, \phi >$ of pseudo-differential operators A relative to eigenfunctions ϕ of the Laplacian on a Riemannian manifold (M, g) with ergodic geodesic flow. QER = quantum ergodic restriction theory has to do with the same problem when the eigenfunctions are restricted to a hypersurface Y in M. Are the restrictions still quantum ergodic? For instance, are restrictions of eigenfunctions of hyperbolic surfaces to closed geodesics ergodic along the closed geodesic? Simple examples show that the answer may be no for some Y. But for generic Y the answer is yes. We give a 'dynamical' condition on Y which is sufficient for QER. We also discuss the QER of the full Cauchy data. In that case, QER holds for any Y.