The Effect of Wind on Wave Shape: Shallow Water

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Onshore (Meisenheimer 2016)

Offshore (Johnson n.d.)

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- Jeffreys (1925), Miles (1957), and Phillips (1957) found growth rates
- Phase-averaged quantities
- Numerical simulations reveal air field
- Simulations often use static wave shape

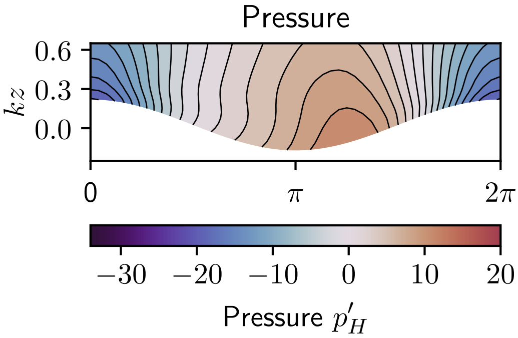


Figure 1: LES simulation of pressure above a wave (Husain et al. 2019).

- Effects of wave shape:
  - Beach morphodynamics
  - Radar altimetry
- Lab measurements of wave shape
  - Leykin et al. (1995)
  - Feddersen and Veron (2005)
- Wave η skewness S and asymmetry A

$$S = \frac{\langle \eta^3 \rangle}{\langle \eta^2 \rangle^{3/2}} \quad \text{and} \quad A = \frac{\langle \mathcal{H}[\eta]^3 \rangle}{\langle \mathcal{H}[\eta]^2 \rangle^{3/2}}$$

  - $\langle \cdot \rangle$ is an average over a wave period and $\mathcal{H}$ is the Hilbert transform

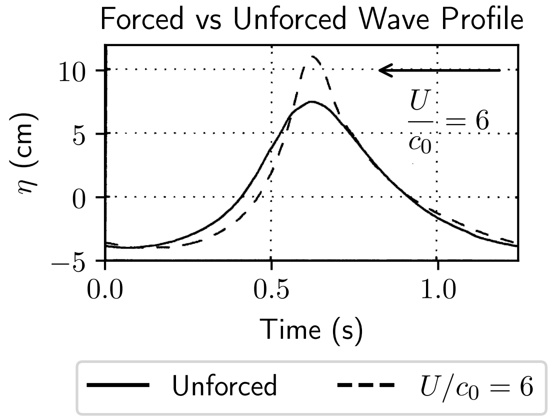
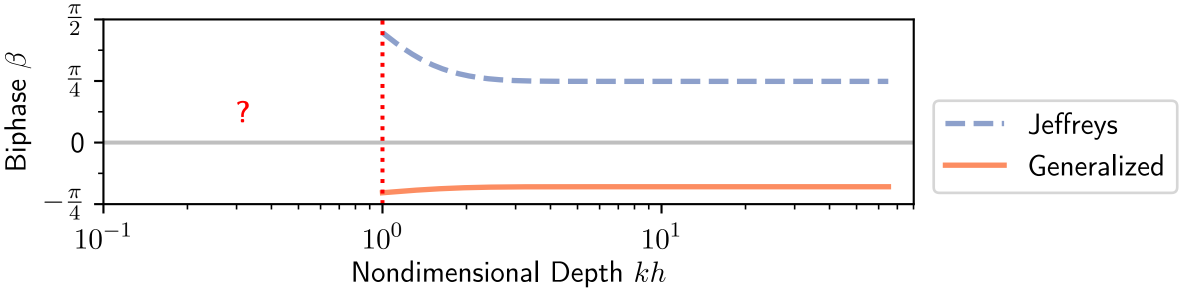


Figure 2: Reproduced from Feddersen and Veron (2005).

- Wind changes biphase β and amplitude A_2 in deep water:

$$\eta k = (ak)\sin[k(x-ct)] + \frac{1}{2}(ak)^2 A_2 \sin[2k(x-ct)+\beta]$$

- Submitted to *J. Fluid Mechanics* (Zdyrski and Feddersen 2019)
- Qualitative agreement with Leykin et al. (1995) experiment
- Larger effect for small kh; limited to $kh \geq 1$



- Incompressible, irrotational, inviscid, 2D flow
- $\eta(x,t)$ and $\nabla\phi(x,t,z) = \vec{u}$
- Pressure enters Bernoulli equation

$$0 = g\eta + \frac{\partial\phi}{\partial t} + \frac{1}{2}\left[\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial z}\right)^2\right] + \frac{p}{\rho_w} \quad \text{at} \quad z = \eta$$

- Unforced waves $p(x,t) = 0$; we need $p(x,t) \neq 0$
- Need to specify pressure profile; choose Jeffreys forcing:

$$p_J(x,t) = P\partial_x\eta(x,t)$$

- Three free, nondimensional parameters:
  - a/h (amplitude)
  - kh (depth)
  - $Pk/(\rho_w g)$ (pressure magnitude)

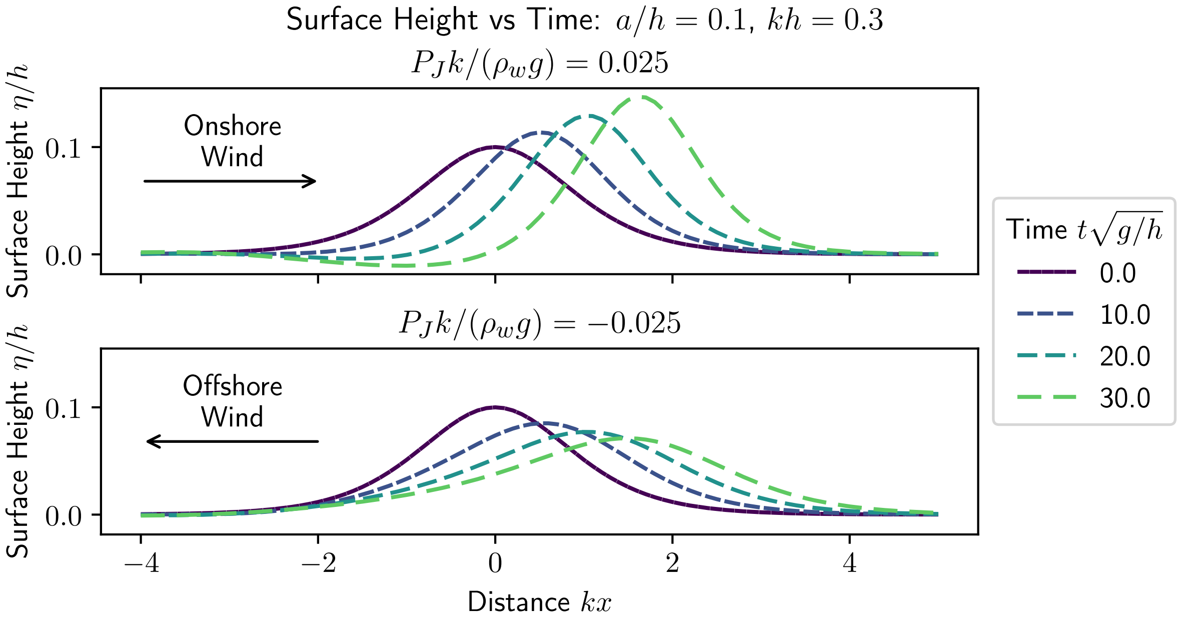
- Assume $\varepsilon := a/h = (kh)^2 = Pk/(\rho_w g) \ll 1$ and a flat bottom
- Method of Multiple Scales
  - $\eta = \varepsilon\eta_1 + \varepsilon^2\eta_2 + \dots$
  - $t_0 = t,\ t_1 = \varepsilon t,\ t_2 = \varepsilon^2 t,\ \dots$
- Multiple scales analysis generates the Korteweg-de Vries (KdV)-Burgers equation

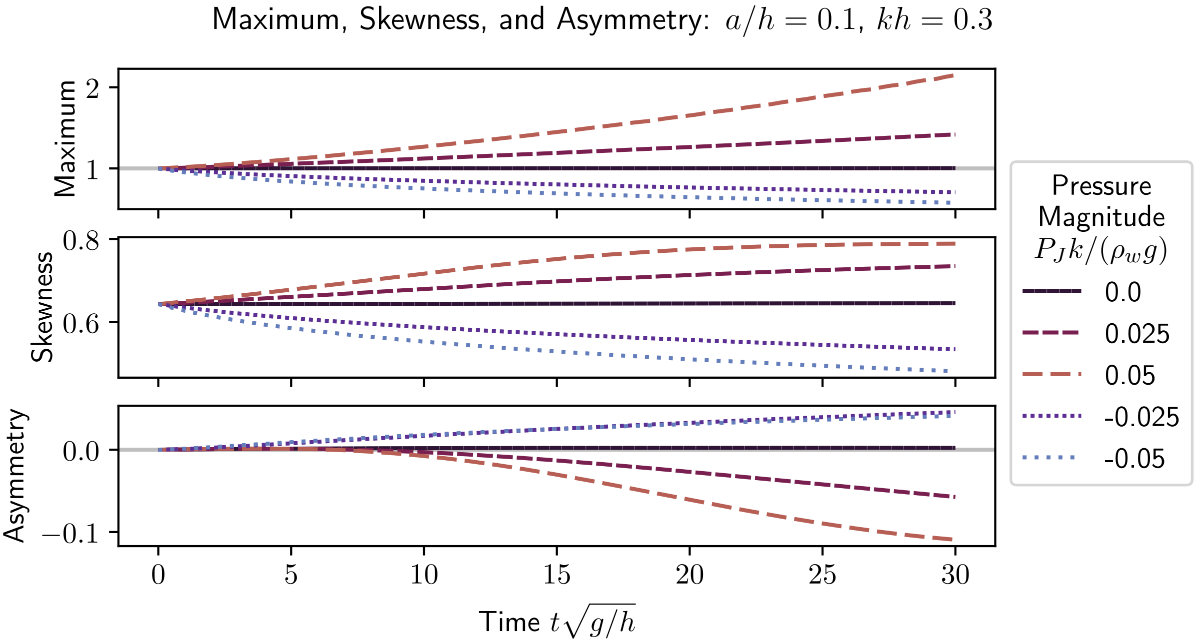
$$\frac{1}{c_0}\frac{\partial\eta_1}{\partial t_1} + \frac{3}{2}\frac{\eta_1}{a}\frac{\partial\eta_1}{\partial x} + \frac{1}{6k^2}\frac{\partial^3\eta_1}{\partial x^3} = -\frac{P}{\rho_w g}\frac{1}{2}\frac{\partial^2\eta_1}{\partial x^2}.$$

  with $c_0 = \sqrt{gh}$
- $P = 0$ reduces to the KdV equation
  - Analytic, propagating wave solutions are *cnoidal waves*
  - Solitary waves are limiting case

$$\eta_1 = c_1 \operatorname{sech}^2\left[\sqrt{\frac{3c_1}{4}}(x - \frac{c_1}{2}t_1)\right]$$

    for $c_1 > 0$
- Sign of P depends on wind direction: onshore wind $\implies$ $P > 0$ and growth
- KdV-Burgers has no known periodic, analytic solutions; we can solve numerically


Surface Height vs Time: $a/h = 0.1$, $kh = 0.3$
$P_J k/(\rho_w g) = 0.025$
Onshore Wind
$P_J k/(\rho_w g) = -0.025$
Offshore Wind
Surface Height η/h
0.0
0.1
−4
−2
0
2
4
Distance kx
Time $t\sqrt{g/h}$
0.0
10.0
20.0
30.0



Maximum, Skewness, and Asymmetry: $a/h = 0.1$, $kh = 0.3$
Maximum
2
1
Skewness
0.8
0.6
Asymmetry
0.0
−0.1
0
5
10
15
20
25
30
Time $t\sqrt{g/h}$
Pressure Magnitude $P_J k/(\rho_w g)$
0.0
0.025
0.05
-0.025
-0.05

- Coupled surface pressure to the Bernoulli equation
- Method of Multiple Scales produced KdV-Burgers equation
- Numerically calculated shape changes consistent with casual observations
- Derived wind-induced maximum, skewness, and asymmetry
- **Surface pressure yields appreciable wave shape changes in shallow water**

Future Work:

- Extend results to periodic waves
- Include dynamic wind-wave coupling

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