The Effect of Wind on Shoaling Wave Shape

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Onshore (Feddersen *et al*. in prep.)

Offshore (Feddersen *et al*. in prep.)

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Wind and Ocean Waves

- Wind causes growth (Jeffreys 1925; Miles 1957; Phillips 1957)
- Pressure and wave phase difference pumps energy
- Simplest model for pressure p (Jeffreys 1925)

$$p \propto \frac{\partial \eta}{\partial x} \quad \text{at} \quad z = \eta$$

  with η wave surface

- Growth know, shape change new (Zdyrski and Feddersen 2020, in JFM)

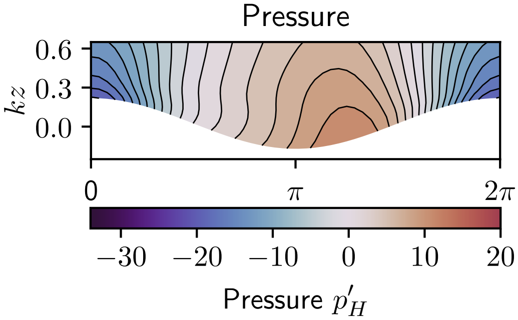


Figure 1: LES simulation of pressure above a wave (Husain et al. 2019).

Wind and Wave Shape

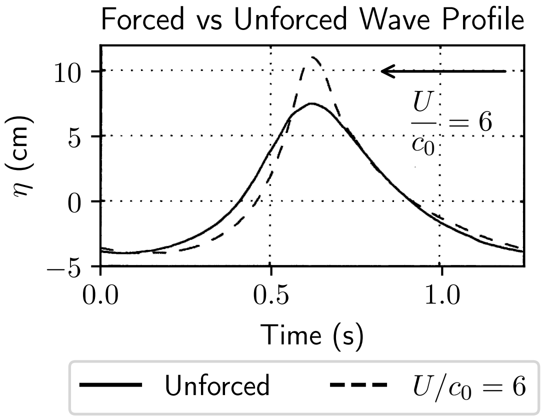


Figure 2: Reproduced from Feddersen and Veron (2005).

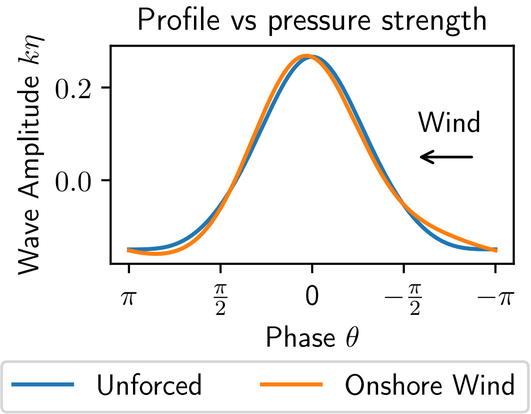


Figure 3: Reproduced from Zdyrski and Feddersen (2020).

Wave Shoaling

- Decreasing water depth (shoaling) also changes wave shape (Elgar and Guza 1985)
- Changes skewness S and asymmetry A

$$S = \frac{\langle \eta^3 \rangle}{\langle \eta^2 \rangle^{3/2}} \quad \text{and} \quad A = \frac{\langle \mathcal{H}[\eta]^3 \rangle}{\langle \mathcal{H}[\eta]^2 \rangle^{3/2}}$$

  - $\langle \cdot \rangle$ average over wave period and $\mathcal{H}$ Hilbert transform
- Goal: examine shape changes from wind and shoaling, applicable in the nearshore

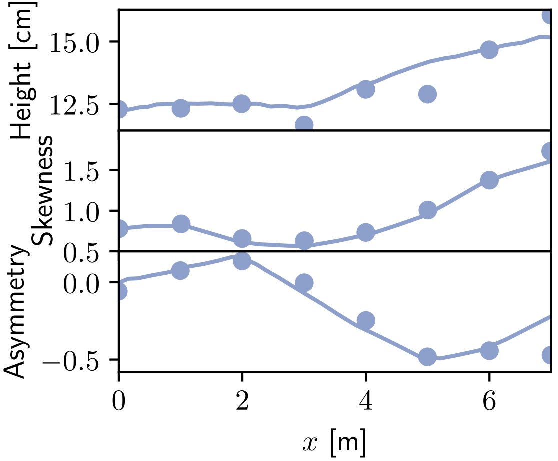


Figure 4: Shape statistics for shoaling, unforced wave experiment; reproduced from Cienfuegos, Barthélemy, and Bonneton (2010).

Setup

- Incompressible, irrotational, inviscid, 2D flow
- $\eta(x,t)$ and $\nabla\phi(x,t,z)=\vec{u}$
- Pressure enters Bernoulli equation

$$0 = g\eta + \frac{\partial \phi}{\partial t} + \frac{1}{2}\left[\left(\frac{\partial \phi}{\partial x}\right)^2 + \left(\frac{\partial \phi}{\partial z}\right)^2\right] + \frac{p}{\rho_w} \quad \text{at} \quad z = \eta$$

- Jeffreys forcing $p_J = P\partial_x\eta(x,t)$
- Assume $\partial_x h \ll 1$, so bottom BC is just $\partial_z\phi = 0$ to leading order
- Four free, nondimensional parameters:
  - a/h_0 (amplitude)
  - kh_0 (depth)
  - $\partial_x h$ (slope)
  - $Pk/(\rho_w g)$ (pressure magnitude)

Mathematics

- Assume $\varepsilon := a/h_0 \sim (kh_0)^2 \sim Pk/(\rho_w g) \ll 1$ and $\partial_x h \sim \varepsilon^{3/2}$
- Method of Multiple Scales
  - $\eta = \varepsilon\eta_1 + \varepsilon^2\eta_2 + \dots$
  - $x_0 = x,\ x_1 = \varepsilon x,\ x_2 = \varepsilon^2 x,\ \dots$
  - Co-moving coordinate $\xi_+ = -t + \int^{x_0} \mathrm{d}x \, / c(x)$
- Variable-coefficient Korteweg-de Vries Burgers equation

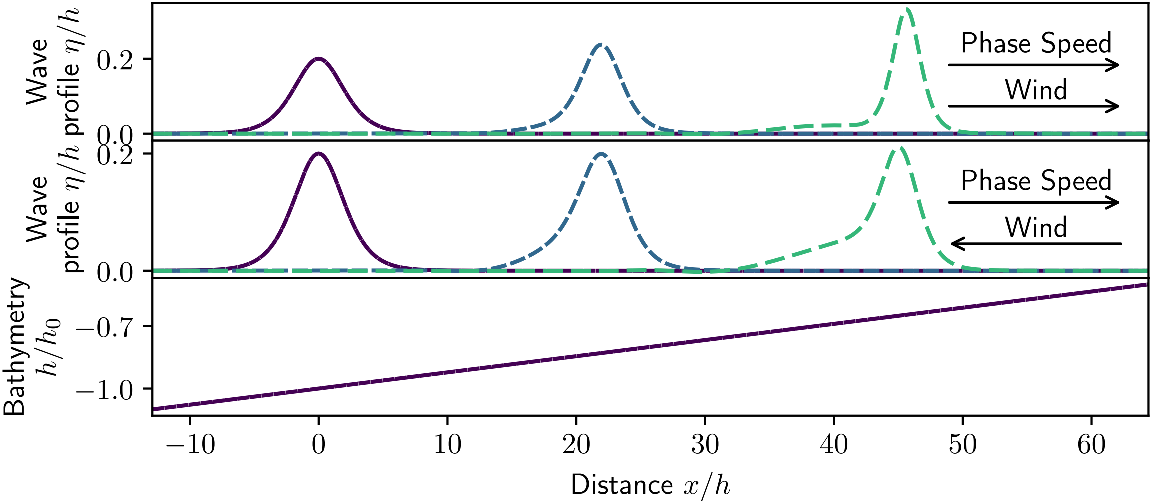
$$\frac{\partial\eta_0}{\partial x_1} + \frac{3}{2}\frac{c_0^3}{c^3(x_1)}\frac{\eta_0}{a_0}\frac{\partial\eta_0}{\partial\xi_+} + \frac{1}{6}\frac{h^3}{a_0}\frac{c(x_1)}{c_0}\frac{\partial^3\eta_0}{\partial\xi_+^3} + \frac{1}{2c}\frac{\partial c(x_1)}{\partial x_1}\eta_0 = -\frac{1}{2}\frac{P}{\rho_w g}\frac{c_0^2}{c^2(x_1)}\frac{\partial^2\eta_0}{\partial\xi_+^2}\,.$$

  with $c(x_1) = \sqrt{gh(x_1)}$ and $c_0 = \sqrt{gh_0}$
- Solitary waves initial conditions

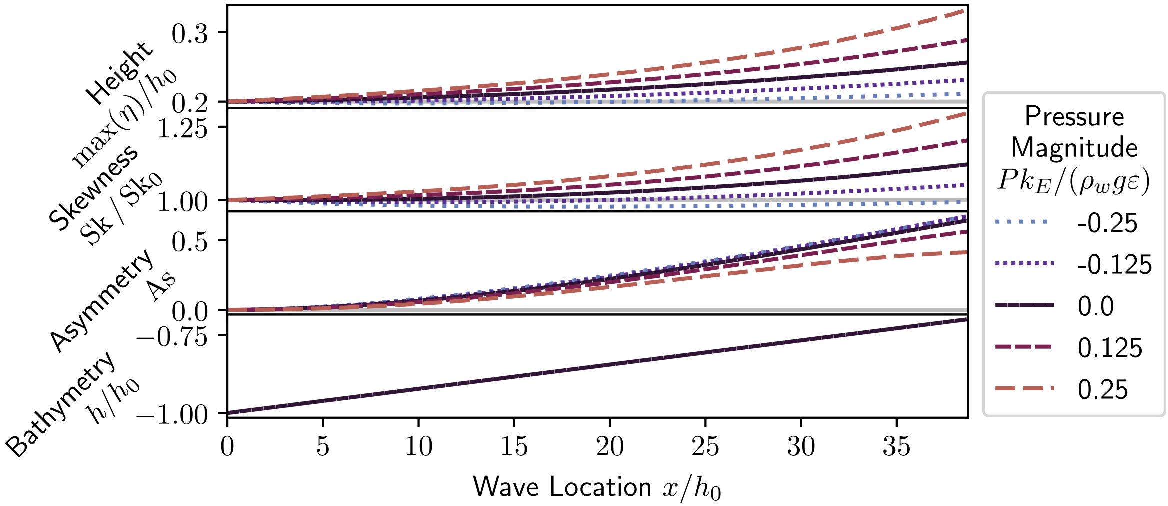
$$\eta_0\Big|_{x_1=0} = 2\,\mathrm{sech}^2\left[\sqrt{\frac{3}{2}}\xi_+\right]$$

- Sign of P depends on wind direction: onshore wind $\implies P > 0$ and growth
- Solve numerically with RK3 central difference scheme and $h(x_1)/h_0 = 1 - 0.1kx_1$

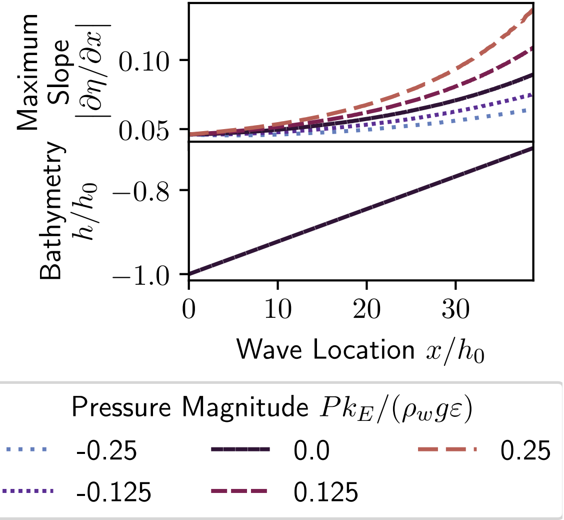
Results: Profile



Results: Height, Skewness, and Asymmetry



Results: Steepness and Breaking



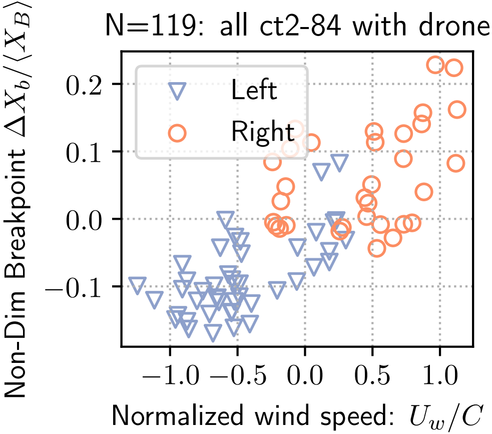


Figure 5: Break point data *vs.* wind speed collected at the Surf Ranch, CA (Feddersen *et al.* in prep.)

Discussion and Conclusion

- Coupled surface pressure to the Bernoulli equation with gently sloping bottom
- Method of Multiple Scales produced damped, variable-coefficient KdV-Burgers equation
- Numerically calculated shape changes consistent unforced shoaling waves
- Derived wind-induced height, skewness, and asymmetry
- **Onshore wind causes steepening in deeper water, while offshore wind delays steepening to shallower water**

Future Work:

- Extend results to periodic waves
- Include dynamic wind-wave coupling

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Cienfuegos, Rodrigo, Eric Barthélemy, and Philippe Bonneton (2010). "Wave-breaking model for Boussinesq-type equations including roller effects in the mass conservation equation". In: *Journal of waterway, port, coastal, and ocean engineering* 136.1, pp. 10–26.

Elgar, Steve and RT Guza (1985). "Observations of bispectra of shoaling surface gravity waves". In: *Journal of Fluid Mechanics* 161, pp. 425–448.

Feddersen, Falk and Fabrice Veron (2005). "Wind effects on shoaling wave shape". In: *Journal of physical oceanography* 35.7, pp. 1223–1228.

Husain, Nyla T et al. (2019). "Boundary layer turbulence over surface waves in a strongly forced condition: LES and observation". In: *Journal of Physical Oceanography* 49.8, pp. 1997–2015.

Jeffreys, Harold (1925). "On the formation of water waves by wind". In: *Proc. R. Soc. Lond. A* 107.742, pp. 189–206.

Miles, John W (1957). "On the generation of surface waves by shear flows". In: *Journal of Fluid Mechanics* 3.2, pp. 185–204.

Phillips, Owen M (1957). "On the generation of waves by turbulent wind". In: *J. Fluid Mech* 2.5, pp. 417–445.

Zdyrski, Thomas and Falk Feddersen (2020). "Wind-Induced Changes to Surface Gravity Wave Shape in Deep to Intermediate Water". In: *Journal of Fluid Mechanics* 903.A31. DOI: 10.1017/jfm.2020.628. arXiv: 1911.07879 [physics.flu-dyn].

Part I

Appendix

Laplace Equation and Boundary Conditions

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \tag{1}$$

$$\left.\frac{\partial \phi}{\partial z}\right|_{z=-\infty} = 0 \tag{2}$$

$$\left.\frac{\partial \phi}{\partial z}\right|_{z=\eta} = \frac{\partial \eta}{\partial t} + \frac{\partial \eta}{\partial x} \left.\frac{\partial \phi}{\partial x}\right|_{z=\eta} \tag{3}$$

$$0 = g\eta + \left.\frac{\partial \phi}{\partial t}\right|_{z=\eta} + \frac{1}{2}\left.\left(\left(\frac{\partial \phi}{\partial x}\right)^2 + \left(\frac{\partial \phi}{\partial z}\right)^2\right)\right|_{z=\eta} + \frac{p}{\rho_w g} \tag{4}$$

with

$$\vec{u} = \boldsymbol{\nabla}\phi \tag{5}$$

Additional Equations

Constraints:

- Periodic[1]

$$\vec{u}(x, z, t) = \vec{u}(x + L, z, t)$$

- Progressive

$$\vec{u}(x, z, t) = \vec{u}'(x - \tau(t), z)$$

- No current $\langle \vec{u} \rangle = 0$

[1] Note: this precludes sloping bottom topographies

Wave Slope

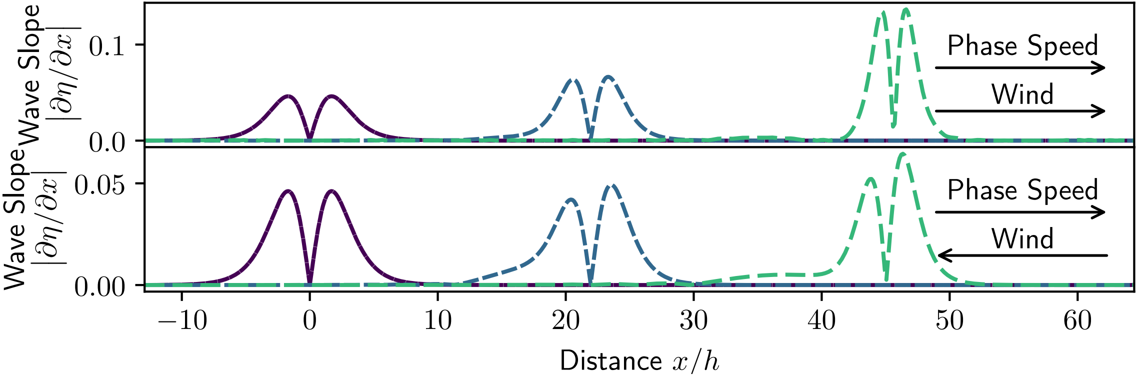

Figure 6: Magnitude of wave slope as a function of distance x/h_0 for (a) onshore and (b) offshore winds.