The Effect of Wind on Shoaling Wave Shape

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Onshore (Feddersen et al. in prep.)

Offshore (Feddersen et al. in prep.)

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Wave Shape and Shoaling

- Effects of wave shape:
  - Beach morphodynamics
  - Radar altimetry
- Decreasing water depth (shoaling) causes wave growth and shape change (Elgar and Guza 1985)
- Miles (1979) derived shoaling-induced shelf for solitary waves
  - Continuously generated on slope
  - Strongest generation in deep water

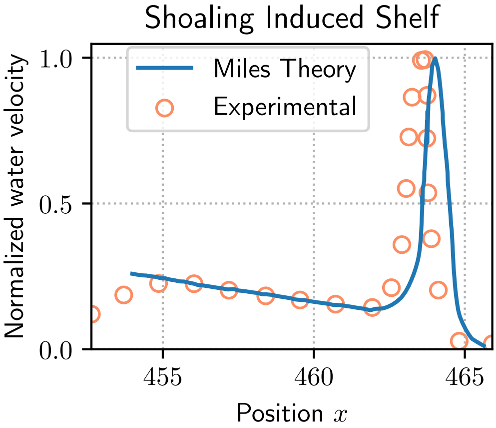


Figure 1: Reproduced from Knickerbocker and Newell (1985).

Wind and Wave Shape

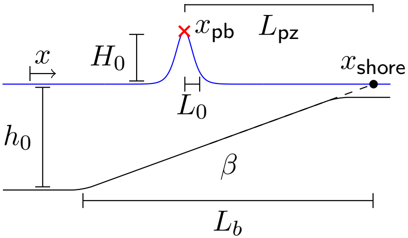
- Wind causes growth (Jeffreys 1925; Miles 1957; Phillips 1957)
- Few experiments on wave shape (Leykin et al. 1995; Feddersen and Veron 2005)
- Flat-bottom theory (Zdyrski and Feddersen 2020, 2021) laid groundwork for shoaling
- Kelly Slater Surf Ranch wave pool for wind on realistic shoaling waves
- **Goal: Investigate how wind affects shoaling wave shape**

Figure 2: Onshore (Feddersen et al. in prep.)

Figure 3: Offshore (Feddersen et al. in prep.)

Setup and Definition of Pre-Breaking

- Convective breaking: surface water moves faster than wave ($\mathrm{Fr} := u/c > 1$, Brun and Kalisch 2018)
- Extended their analysis by calculating Fr for wind-forced waves on sloping bottoms
- Our approximations require small u: stop at "pre-breaking" $\max_x(\mathrm{Fr}) = 1/3$



- Periodic domain with initial depth h_0
- Bathymetry smoothly transitions to slope β
- Beach width L_b with shallow plateau
- Solitary wave: height H_0 and width L_0
- Pre-breaking zone width L_{pz} is distance from pre-breaking point x_{pb} to shoreline x_{shore}

Mathematics

- Simplest model for surface pressure $p = P\,\partial\eta/\partial x$ (Jeffreys 1925)
- Parameters: • H_0/h_0 (wave height) • h_0/L_0 (wave width)
  • L_0/L_b (beach width) • $P/(\rho_w g L_0)$ (pressure magnitude)
- Assume $\varepsilon_0 := H_0/h_0 \sim (h_0/L_0)^2 \sim P/(\rho_w g L_0) \sim L_0/L_b \ll 1$
- Method of Multiple Scales: • $\eta = \varepsilon_0\eta_1 + \varepsilon^2\eta_2 + \ldots$ • $x_0 = x$, $x_1 = \varepsilon_0 x$, ...
- Variable-coefficient Korteweg–de Vries–Burgers equation

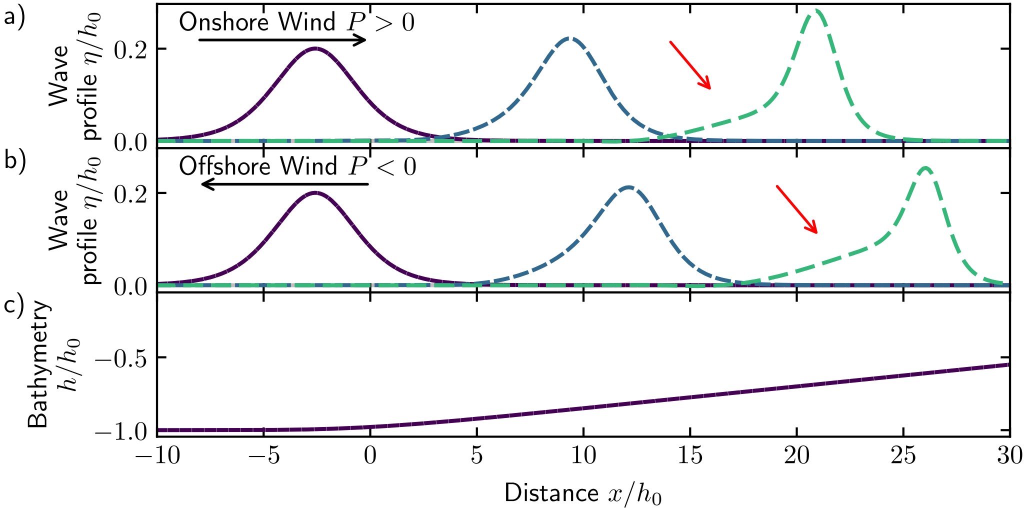
$$\frac{1}{c}\frac{\partial\eta}{\partial t} + \frac{\partial\eta}{\partial x} + \frac{1}{2}\frac{\partial c}{\partial x}\frac{\eta}{c} + \frac{3}{2}\frac{c_0^2}{c^2}\frac{\eta}{h_0}\frac{\partial\eta}{\partial x} + \frac{1}{6}h_0^2\frac{c^4}{c_0^4}\frac{\partial^3\eta}{\partial x^3} = -\frac{1}{2}\frac{P}{\rho_w g}\frac{\partial^2\eta}{\partial x^2}.$$

  with $c = \sqrt{gh(x)}$ and $c_0 = \sqrt{gh_0}$
- Solitary waves initial condition

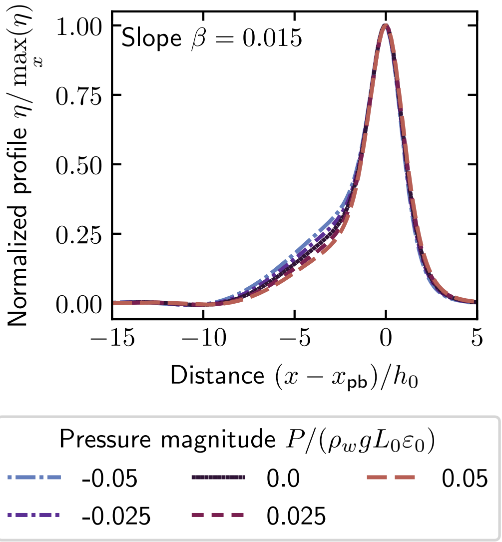
$$\left.\frac{\eta}{h_0}\right|_{t=0} = \varepsilon_0\,\mathrm{sech}^2\left[\sqrt{\frac{3\varepsilon_0}{4}}\frac{x}{h_0}\right]$$

- Solve numerically with RK3 central difference scheme

Results: Profile



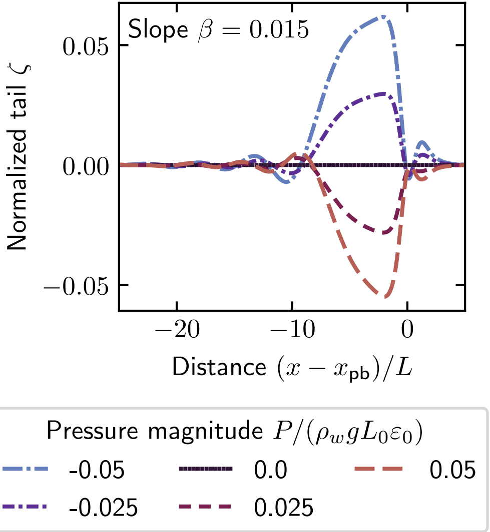
Results: Pre-Breaking Wave Shape



- Unforced $P = 0$ line described by Miles shoaling theory
  - Superposition of sech^2 solitary wave and shoaling-induced shelf
- vcKdV–Burgers can be re-written as ccKdV = shoaling + wind-forcing
  - Shoaling term coefficient $\beta/(\varepsilon_0 h_0/L_0) \approx 0.2$
  - Wind-forcing term coefficient $P/(\rho_w g L_0 \varepsilon_0) = 0.05$
  - Act as perturbations to KdV sech^2
- Hypothesis: Full solution is a linear superposition of KdV sech^2, Miles shelf, and bound, dispersive tail

Results: Wind-Induced Bound, Dispersive Tail

- Subtract $P = 0$ solution to remove sech^2 and Miles shelf
- Enhances/suppresses shoaling-induced shelf
- Matches bound, dispersive tail for flat-bottom wind forcing
- shoaling, wind-forced soliton = KdV sech^2
  + Miles shelf
  + wind-induced bound, dispersive tail



Summary

- Coupled surface pressure to shoaling solitary waves on a gently sloping bottom
- Method of Multiple Scales produced variable-coefficient KdV–Burgers equation
- Shoaling and wind-forcing are perturbations to sech^2
- **Wind can affect shoaling wave shape in shallow water**
- In review: Journal of Fluid Mechanics (arXiv:2110.05519 [physics.flu-dyn])

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Appendix

Laplace Equation and Boundary Conditions

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \tag{1}$$

$$\left.\frac{\partial \phi}{\partial z}\right|_{z=-h} = -\frac{\partial h}{\partial x} \tag{2}$$

$$\left.\frac{\partial \phi}{\partial z}\right|_{z=\eta} = \frac{\partial \eta}{\partial t} + \frac{\partial \eta}{\partial x} \left.\frac{\partial \phi}{\partial x}\right|_{z=\eta} \tag{3}$$

$$0 = g\eta + \left.\frac{\partial \phi}{\partial t}\right|_{z=\eta} + \frac{1}{2} \left.\left(\left(\frac{\partial \phi}{\partial x}\right)^2 + \left(\frac{\partial \phi}{\partial z}\right)^2 \right)\right|_{z=\eta} + \frac{p}{\rho_w g} \tag{4}$$

with

$$\vec{u} = \boldsymbol{\nabla} \phi \tag{5}$$

Additional Equations: Deep Water

Constraints:

- Periodic[1]

$$\vec{u}(x, z, t) = \vec{u}(x + L, z, t)$$

- Progressive

$$\vec{u}(x, z, t) = \vec{u}'(x - \tau(t), z, t)$$

- No current $\langle \vec{u} \rangle = 0$

[1] Note: this precludes sloping bottom topographies

Additional Equations: Shallow Water

Constraints:

- Localized

$$\eta, \vec{u} \to 0 \quad \text{as} \quad |x|$$

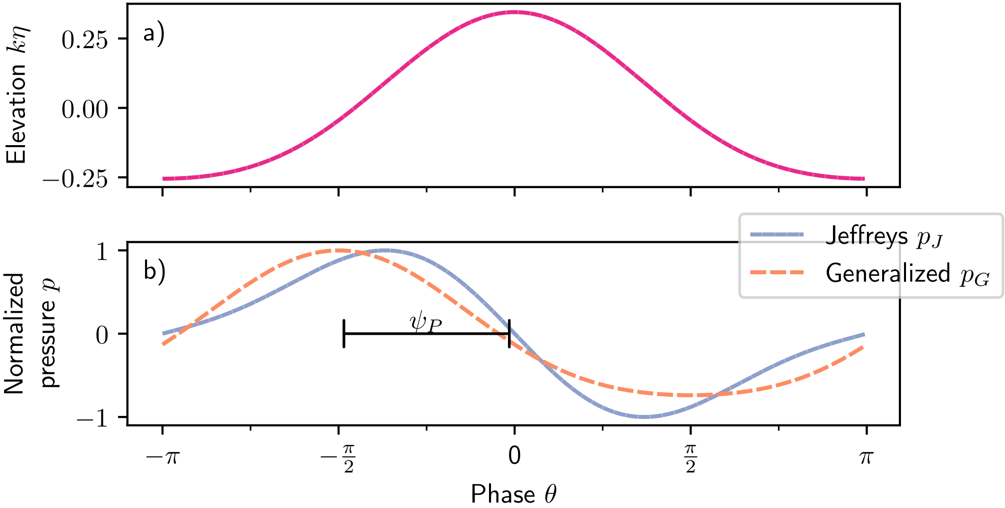
- Progressive

$$\vec{u}(x, z, t) = \vec{u}'(x - \tau(t), z, t)$$

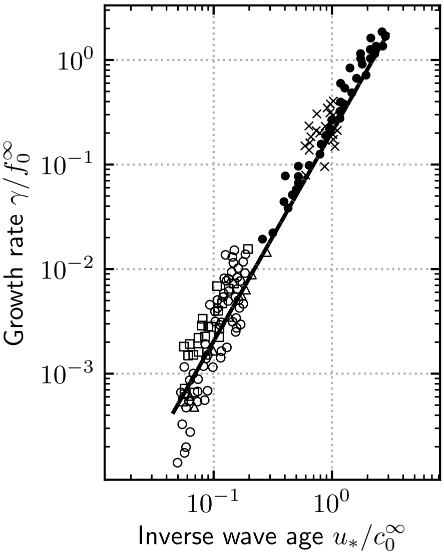
- No current at bottom

$$\vec{u} = 0 \quad \text{at} \quad z = -h$$

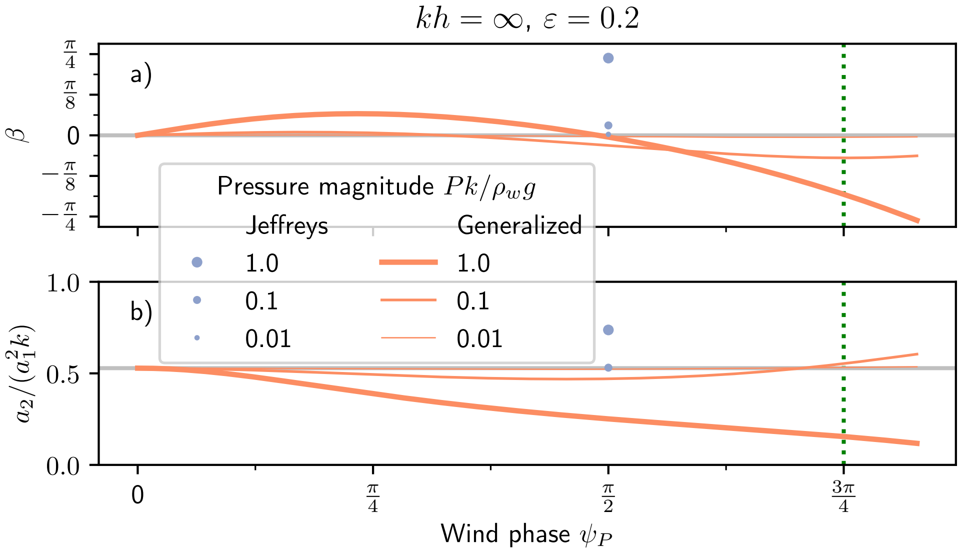
Forcing Types



Wind Speed vs. Growth Rate (Komen et al. 1994)

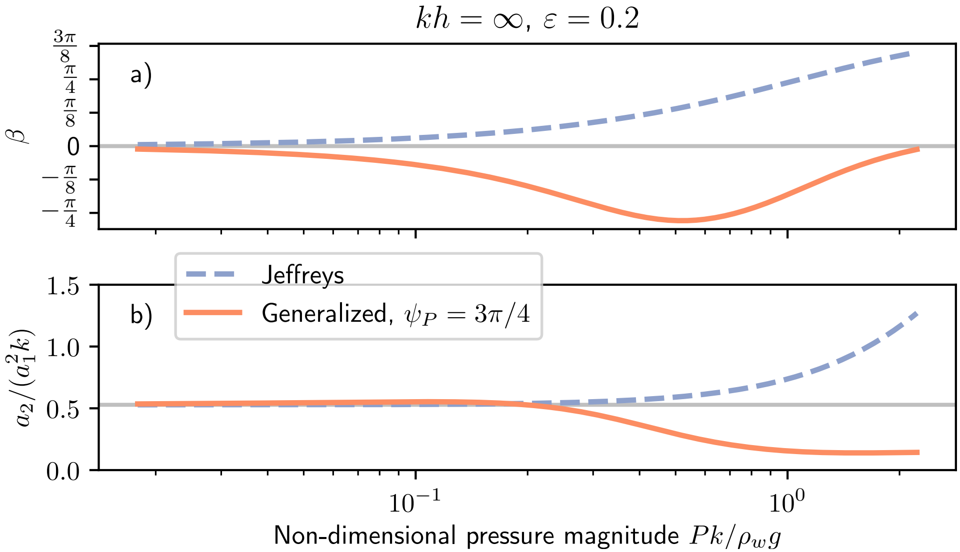


Wind Phase

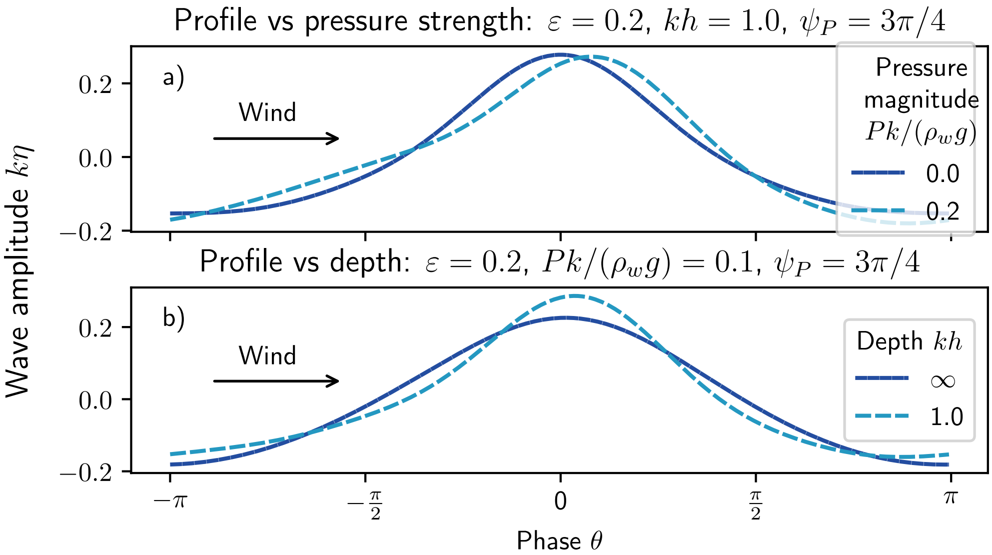


$kh = \infty$, $\varepsilon = 0.2$
a)
b)
β
$\frac{\pi}{4}$
$\frac{\pi}{8}$
0
$-\frac{\pi}{8}$
$-\frac{\pi}{4}$
$a_2/(a_1^2 k)$
1.0
0.5
0.0
0
$\frac{\pi}{4}$
$\frac{\pi}{2}$
$\frac{3\pi}{4}$
Wind phase ψ_P
Pressure magnitude $Pk/\rho_w g$
Jeffreys
Generalized
1.0
0.1
0.01
1.0
0.1
0.01

Pressure Magnitude

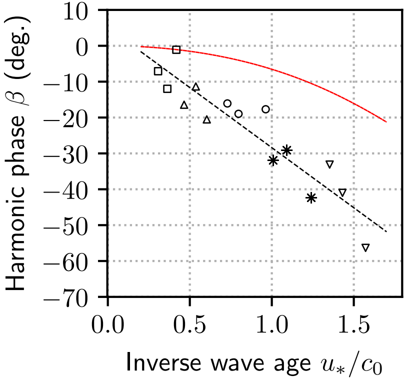


Depth



Profile vs pressure strength: $\varepsilon = 0.2,\ kh = 1.0,\ \psi_P = 3\pi/4$
a)
Wind
Pressure magnitude $Pk/(\rho_w g)$
0.0
0.2
Profile vs depth: $\varepsilon = 0.2,\ Pk/(\rho_w g) = 0.1,\ \psi_P = 3\pi/4$
b)
Wind
Depth kh
∞
1.0
Wave amplitude $k\eta$
Phase θ
0.2
0.0
-0.2
$-\pi$
$-\frac{\pi}{2}$
0
$\frac{\pi}{2}$
π

Comparison to Leykin et al. (1995)



Wave Shape (Feddersen and Veron 2005)

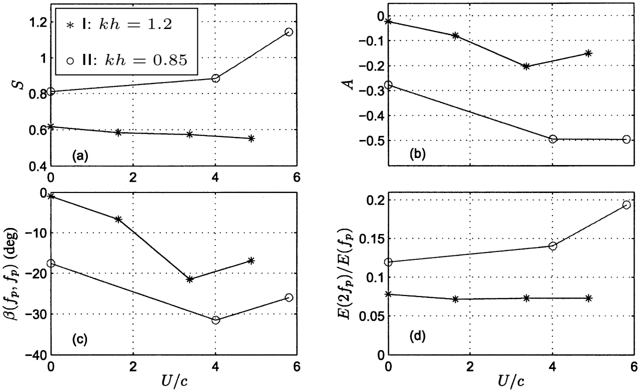
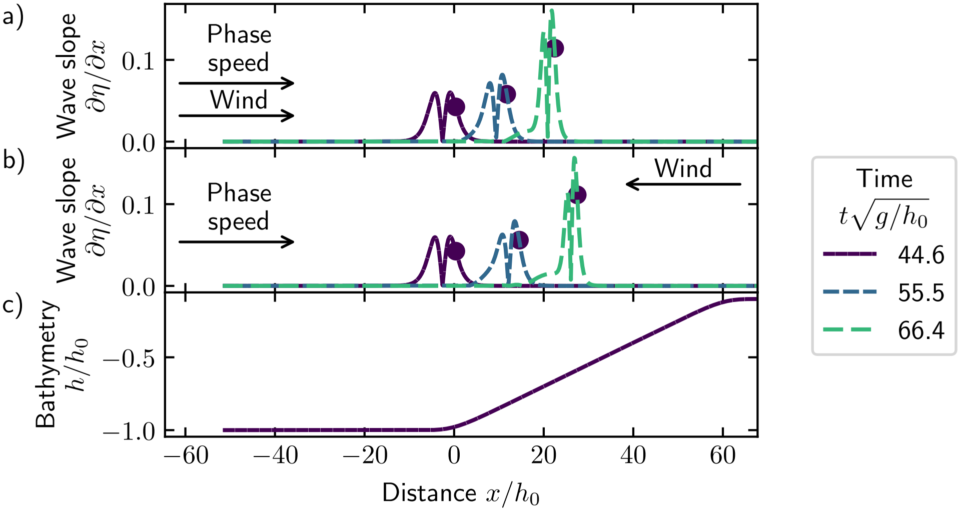


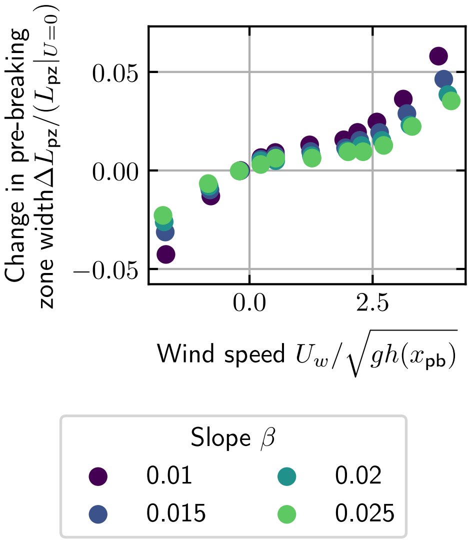
Figure 4: (a) Skewness, (b) asymmetry, (c) biphase, and (d) energy of shoaling waves under the influence of wind (Feddersen and Veron 2005).

Wave Slope



a)
Wave slope
$\partial\eta/\partial x$
0.1
0.0
Phase speed
Wind
b)
Wave slope
$\partial\eta/\partial x$
0.1
0.0
Phase speed
Wind
c)
Bathymetry
h/h_0
-0.5
-1.0
-60
-40
-20
0
20
40
60
Distance x/h_0
Time
$t\sqrt{g/h_0}$
44.6
55.5
66.4

Results: Pre-Breaking Location



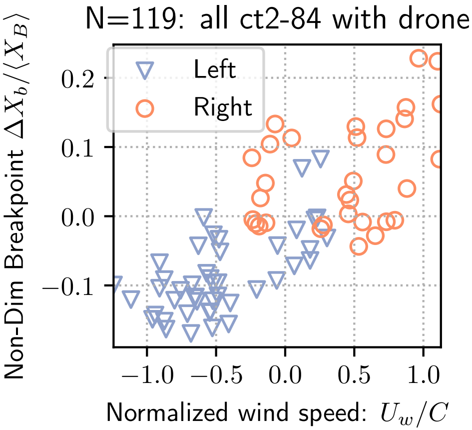


Figure 5: Break point data vs. wind speed collected at the Kelly Slater Surf Ranch, CA (Feddersen et al. in prep.)