Dissipative Solitons in Graphene

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Key Points

- Perturbation techniques used to derive solitons in the Dirac fluid and Fermi liquid regimes of graphene
- Dissipative effects shown to cause solitons to slow and decay according to the Korteweg-de Vries-Burgers equation
- Background current included, allowing for new experiments to measure electron viscosity

Introduction: Solitons

- Localized disturbances propagating steadily
- Balance of **nonlinear focusing** and **dispersion**
- Prototype: Korteweg-de Vries (KdV) equation

$$\frac{\partial f}{\partial t} + f\frac{\partial f}{\partial x} + \frac{\partial^3 f}{\partial x^3} = 0$$

- 1-parameter (a) family of solutions

$$f(x,t) = a\,\mathrm{sech}^2\left(\sqrt{\frac{a}{12}}\left[x - t\frac{a}{3}\right]\right) \qquad (1)$$

Introduction: Hydrodynamic Regime

Graphene is a two-dimensional sheet of carbon atoms that can be made pure enough to have a hydrodynamic regime. In fact, graphene has two hydrodynamic regimes: the low temperature, high voltage **Fermi liquid** regime $\mu \gg k_B T$, and the low voltage, high temperature **Dirac fluid** regime $k_B T \gg \mu$.

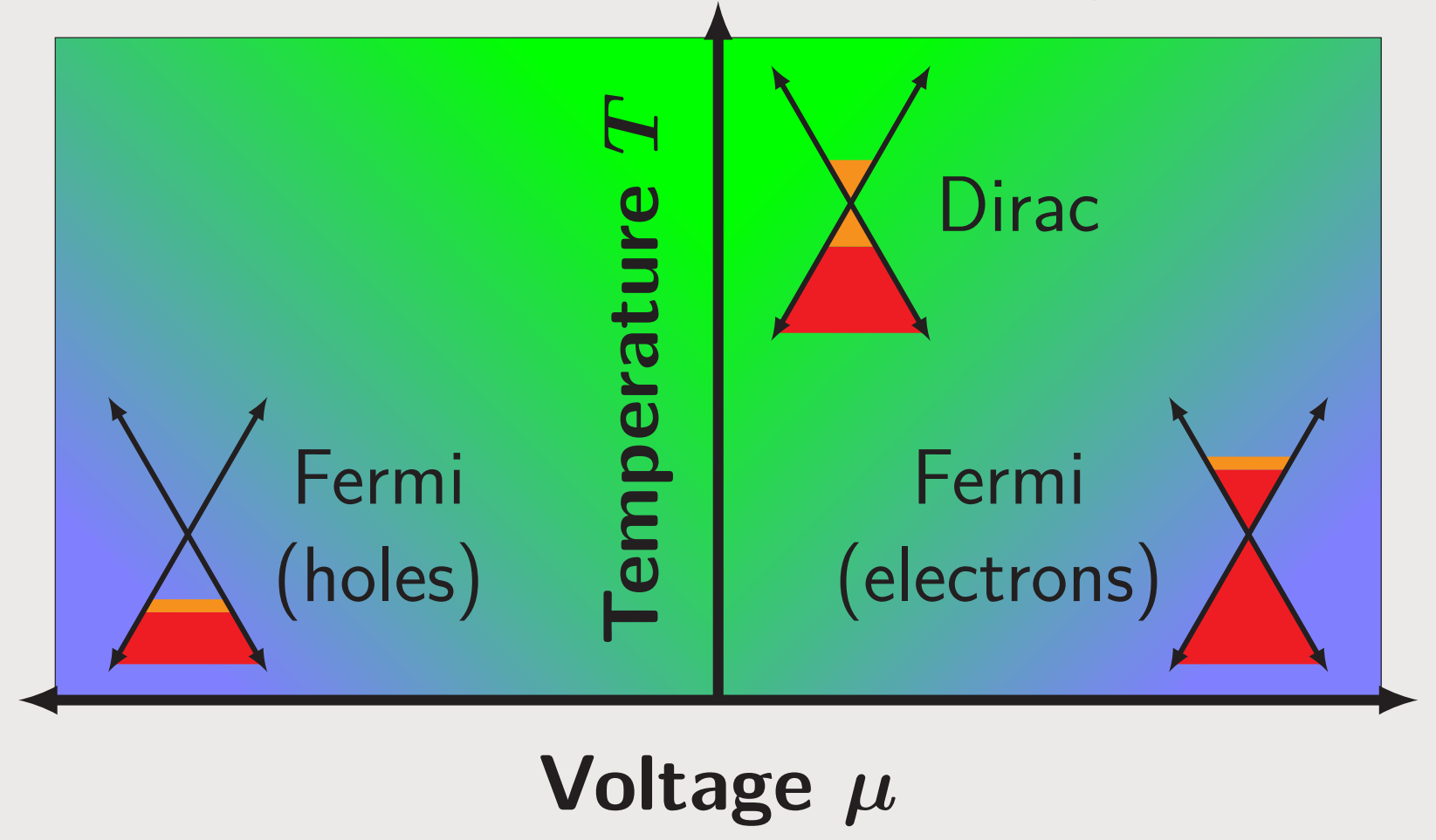

Figure 1: Phase diagram of the Fermi liquid and Dirac fluid regimes of graphene. Inset figures depict dispersion cones with completely filled momentum modes and thermally excited momentum modes. Reproduced from Lucas and Fong [1].

In these hydrodynamic regimes, graphene has a large viscosity—more than 10x that of honey[2]. Some measurements have been made in the Fermi regime[2], but data in the Dirac regime is lacking. Here, we propose **viscometry experiments** using solitons that are applicable in both the **Dirac and Fermi regimes**.

Setup

Restricting to one-dimensional propagation, we will be solving for the **charge carrier density n**, the **fluid velocity u**, the **pressure P** and the **energy ϵ**.

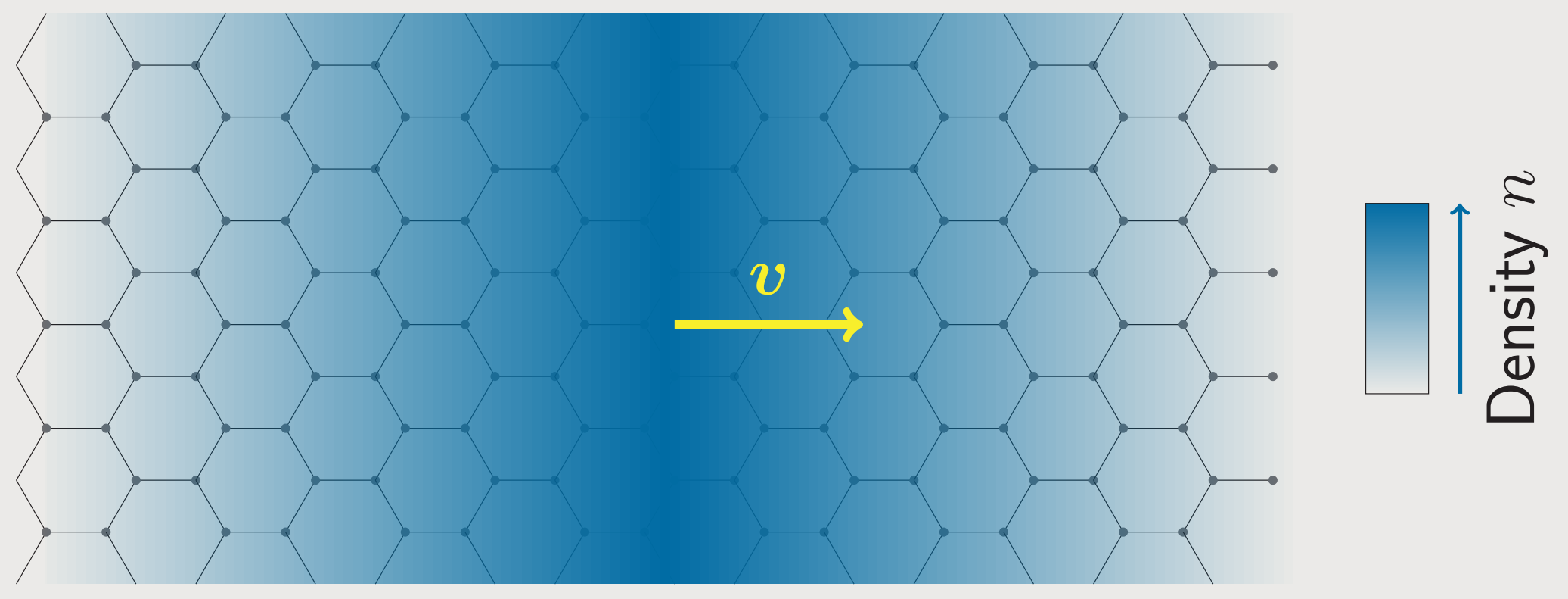

Figure 2: One-dimensional density wave propagation in graphene with velocity v.

Conducting plates (gates) are placed above/below the graphene sheet a distance d to make the electrostatic interaction short-ranged. The space between the graphene and the gates is filled with a **dielectric** κ.

$$E = \frac{2\pi e^2 d}{\kappa}\left(1 + d^2\frac{\partial^2}{\partial x^2}\right)\frac{\partial n}{\partial x} + \mathcal{O}(d\partial_x)^4$$

Governing Equations

- Charge Conservation:

$$\frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = \frac{\sigma_Q}{e^2}\frac{\partial}{\partial x}\left[k_B T\frac{\partial}{\partial x}\left[\frac{\mu}{k_B T}\right] - e\frac{\partial E}{\partial x}\right] \qquad (2)$$

- Energy Conservation:

$$\frac{\partial \epsilon}{\partial t} + \frac{\partial [u(\epsilon + P)]}{\partial x} = -enEu \qquad (3)$$

- Momentum Conservation:

$$\frac{\partial [u(\epsilon + P)]}{\partial t} + \frac{\partial P}{\partial x} + enE = (\eta + 2\zeta)\frac{\partial^2 u}{\partial x^2} \qquad (4)$$

- Thermodynamic equation of state

Perturbation Expansion

- Expand dependent variables in small parameter ε

$$n = n_0 + \varepsilon n_1 + \varepsilon^2 n_2 + \dots$$

- Multiple Scales Method: introduce slower timescales

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t_0} + \varepsilon\frac{\partial}{\partial t_1} + \varepsilon^2\frac{\partial}{\partial t_2} + \dots$$

- Collect order-by-order in ε and solve for n, giving the KdV-Burgers equation

$$\frac{\partial n_1}{\partial t_1} + \mathcal{A}\frac{\partial n_1}{\partial x} + \mathcal{B}n_1\frac{\partial n_1}{\partial x} + \mathcal{C}\frac{\partial^3 n_1}{\partial x^3} = \mathcal{G}\frac{\partial^2 n_1}{\partial x^2}$$

- The dissipative term $\mathcal{G}$ is a linear combination of the dissipative coefficients σ_Q, η, and ζ

Results

If dissipation is weak, solutions are approximately KdV-type solitons (eq. (1)) with time-dependent a:

$$a(t) = \frac{1}{1 + \varepsilon\frac{t}{t_d}} \quad \text{with} \quad t_d = \frac{45|\mathcal{C}|}{4\mathcal{G}|\mathcal{B}|}$$

This causes three changes: **amplitude decay**, **widening**, and **deceleration**.

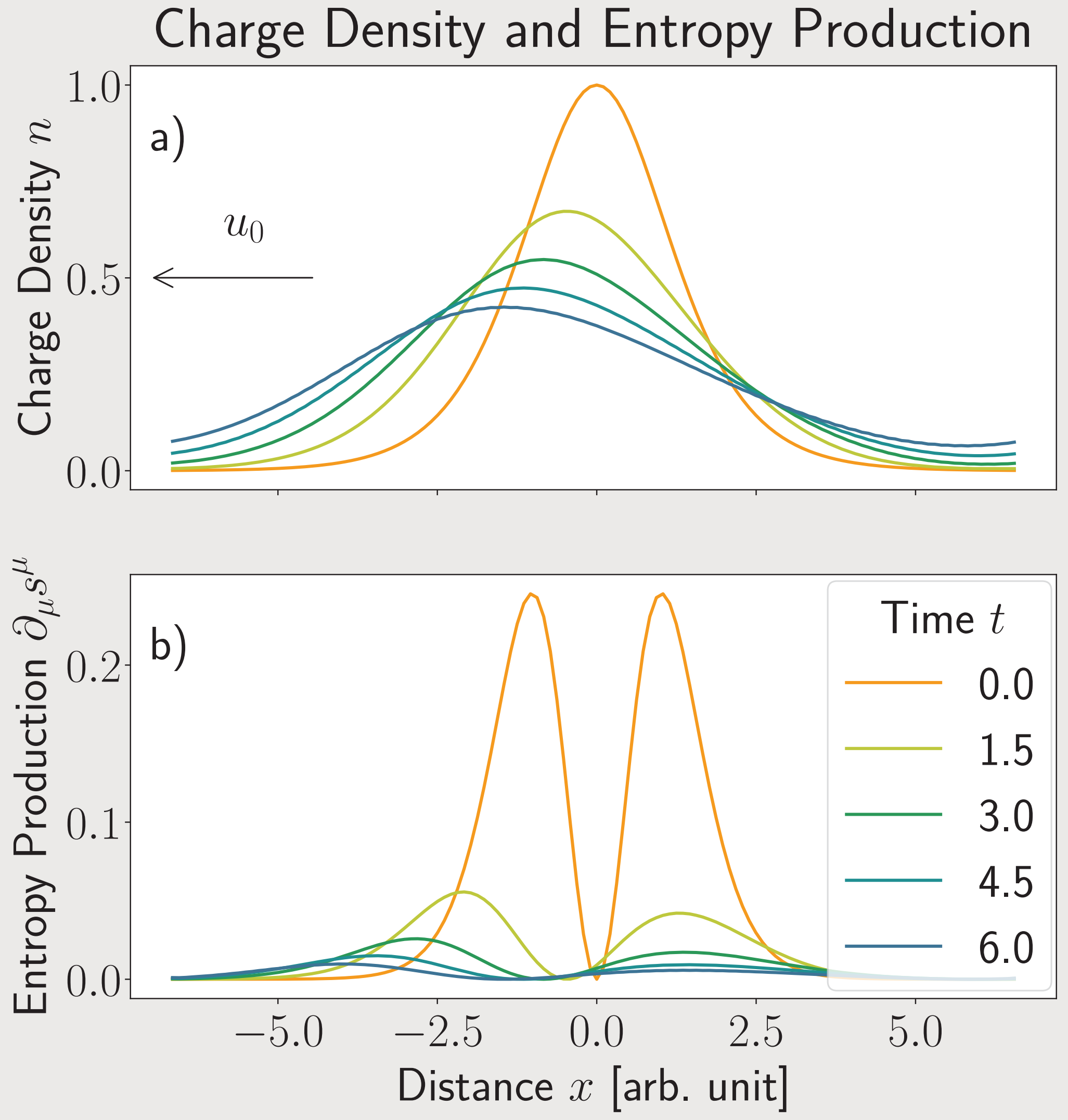

Figure 3: Soliton decay (a) and entropy generation (b) as functions of time. The arrow represents a uniform background current u_0 to counter the initial propagation speed v_0. All quantities are nondimensionalized.

Energy is conserved at this order (eq. (3)). The entropy divergence $\partial_\mu s^\mu$, caused by **spreading**, shows that dissipation is concentrated on the front/rear faces.

Viscometry Proposal: Timing

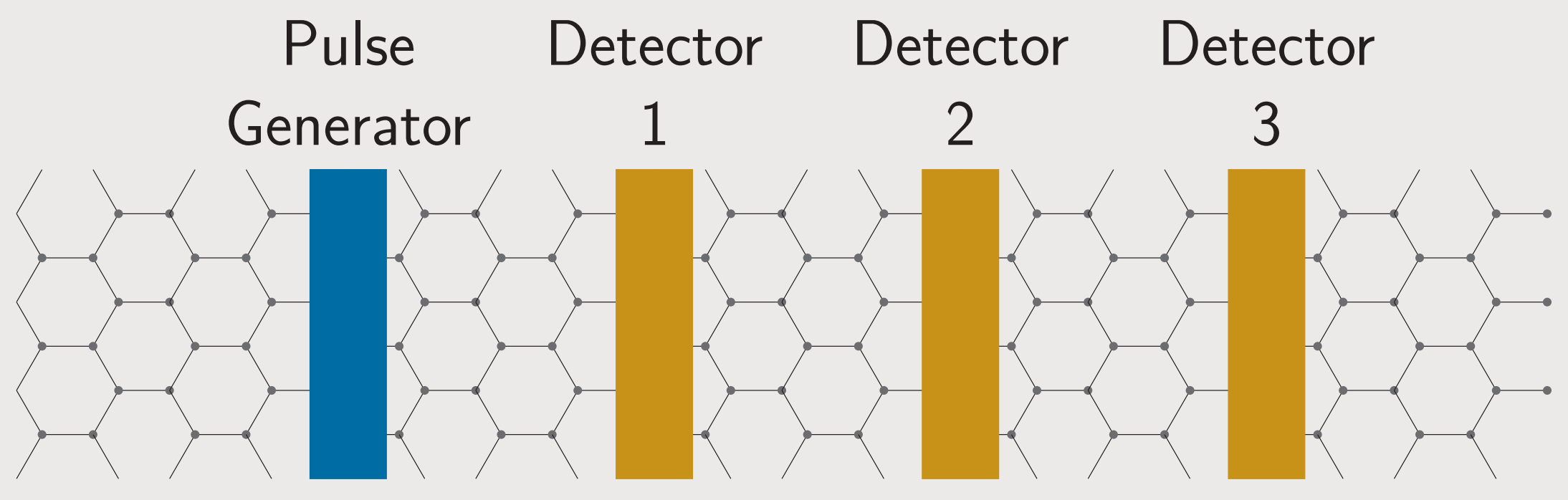

Figure 4: Proposed experiment for measuring soliton deceleration.

- Multiple detectors determine propagation speed
- Deceleration determines dissipation rate
- No background current u_0 means simpler setup
- Larger graphene samples needed for fast soliton propagation ($v \sim c/300$)

Viscometry Proposal: Amplitude

- Direct measurement of amplitude decay also determines dissipation rate
- Large background current u_0 counteracts propagation speed $v \sim c/300$
- Stationary solitons are easier to measure
- Only valid if graphene has free-slip boundary

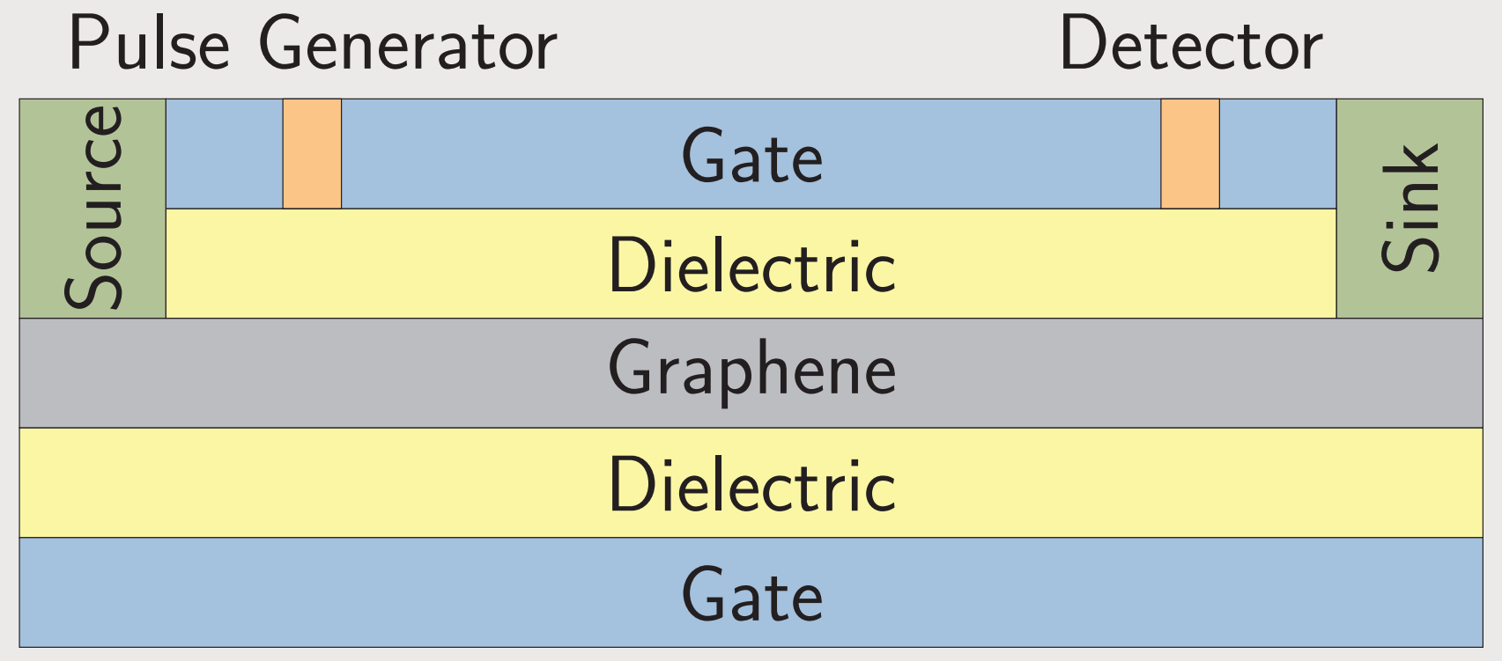

Figure 5: Side view of proposed amplitude experiment.

Summary

- Solitons previously derived for inviscid Fermi regime[3]
- In this work, results have been extended to the viscous Dirac and Fermi regimes
- Inclusion of arbitrary background current allows solitons' propagation speed to be tuned
- Measurements of soliton decay rates or deceleration can yield experimental viscometry data

References

[1] A. Lucas and K. C. Fong, Journal of Physics: Condensed Matter **30**, 053001 (2018).

[2] D. Bandurin, I. Torre, R. K. Kumar, M. B. Shalom, A. Tomadin, A. Principi, G. Auton, E. Khestanova, K. Novoselov, I. Grigorieva, et al., Science **351**, 1055 (2016).

[3] D. Svintsov, V. Vyurkov, V. Ryzhii, and T. Otsuji, Physical Review B **88**, 245444 (2013).

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